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Midterm Exam 2017

Q1. the type of answer is Number

Ans: Jacobian Matrix

$$\begin{vmatrix} \frac{dx}{du} & \frac{dx}{dv} & \frac{dx}{dw} \\ \frac{dy}{du} & \frac{dy}{dv} & \frac{dy}{dw} \\ \frac{dz}{du} & \frac{dz}{dv} & \frac{dz}{dw} \end{vmatrix} = \begin{vmatrix} v & u & 1 \\ w & 1 & u \\ 1 & w & v \end{vmatrix}$$

$= v(1\cdot v - u\cdot w) + (-u)\cdot(w\cdot v - u) + 1(w^2 - 1^2)$

$= \cancel{v^2 - uvw - uvw + u^2 + w^2 - 1}$

$= \cancel{v^2 + u^2 + w^2 - 2uvw - 1}$

$\cancel{= 4 + 4 + 4 - 2\cdot(2\cdot 2\cdot 2) - 1}$

$\cancel{= 12 - 1 - 8}$

$\cancel{= 3}$

$= 2(2-4) - 2(4-2) + 1(4-1)$

$= -4 - 4 + 3$

$= -5$

Q2. (i) ~~the type of ans is~~

$\text{curl}(F) = \nabla \times F$

$$= \begin{vmatrix} i & j & k \\ \frac{d}{dx} & \frac{d}{dy} & \frac{d}{dz} \\ P & Q & R \end{vmatrix}$$

$= i\left(\frac{d}{dy}\cdot R - Q\cdot\frac{d}{dz}\right) - j\left(R\cdot\frac{d}{dx} - P\cdot\frac{d}{dz}\right) + k\left(\frac{d}{dx}\cdot Q - P\frac{d}{dy}\right)$

$= i\left[(x^3 + x - \sin(x+y+z)) - (x^3 + x - \sin(x+y+z))\right]$

$- j\left[(3x^2y + y - \sin(x+y+z)) - (3x^2y + y - \sin(x+y+z))\right]$

$\cancel{+ k\left[(3x^2z + z - \sin(x+y+z)) - (3x^2z + z - \sin(x+y+z))\right]}$

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$= (0,0,0)$

because curl(F) is (0,0,0), thus F is a conservatie vector field.

(ii) the type of ans is multivarible function

Ans: $F = \nabla f$

$f = \int 3x^2yz + yz + \cos(x+y+z)\, dx$

$= x^3yz + xyz + \sin(x+y+z) + g(y,z)$

$f_y = x^3z + xz + \cos(x+y+z) + g_y(y,z)$

$= x^3z + xz + \cos(x+y+z)$

$\therefore g_y(y,z) = 0$

g is not depend on y

$\therefore g(y,z) = h(z)$

$f_z = x^3y + xy + \cos(y+x+z) + h'(z)$

$= x^3y + xy + \cos(x+y+z)$

$\therefore h'(z) = 0$

$h(z) = C$ we can take $C=0$

$\therefore f(x,y,z) = x^3yz + xyz + \sin(x+y+z)$

(iii) the type of ans is Numbers.

$\because$ F is a conservative vector field.

$\therefore \int_C F \cdot dr = f(\sin\pi, \cos\pi + 1, \sin 2\pi) - f(\sin 0, \cos 0 + 1, \sin 0)$

$= f(0,0,0) - f(0,2,0)$

$= -\sin 2$

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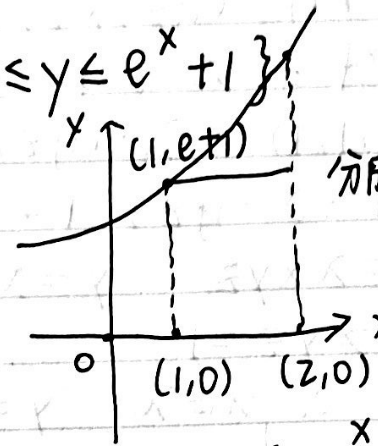
Q3. the type of ans is Sum of 2 abstract double-integrals.

$$\int_1^2 \int_0^{e^x+1} F(x,y)\,dy\,dx$$

Ans: $\{(x,y) \mid 1 \le x \le 2,\ 0 \le y \le e^x+1\}$



分成2个部分看

$y = e^x + 1$

$y - 1 = e^x$

$\ln(y-1) = x$

$x = \ln(y-1)$

type 1: $\{(x,y) \mid 1 < x < 2,\ 0 < y < e^x + 1$

type 2: $\{(x,y) \mid 0 \le y \le e+1,\ 1 \le x \le 2\} \cup$

$\{(x,y) \mid e+1 \le y \le e^2+1,\ \ln(y-1) \le x \le 2.\}$.

$$\therefore \int_0^{e+1} \int_1^2 F(x,y)\,dx\,dy + \int_{e+1}^{e^2+1} \int_{\ln(y-1)}^{2} F(x,y)\,dx\,dy$$

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Q4. the type of ans is Numbers

$f = x+y+z \qquad xyz = 1$

Ans: $\nabla f = (1,1,1) \qquad g = xyz - 1$

$\nabla g = (yz, xz, xy)$

$\nabla f = \lambda \cdot \nabla g$

$(1,1,1) = \lambda \cdot (yz, xz, xy)$

$1 = \lambda \cdot yz, \quad 1 = \lambda \cdot xz, \quad 1 = \lambda \cdot xy$

$x = \lambda xyz \qquad y = \lambda \cdot xyz \qquad z = \lambda \cdot xyz$

$\because xyz = 1$

$\therefore x = \lambda \quad y = \lambda \quad z = \lambda$

$x \cdot y \cdot z = \lambda^3$

$1 = \lambda^3$

$\lambda = 1$

$\therefore$ the location is $(1,1,1)$

~~fct~~ $f = x+y+z$

$= 1+1+1 = 3$

$\therefore$ the smallest value is 3.

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Q5. the type of ans is Number

$$\iiint_E 48xyz\, dV$$

$\{(x,y,z) \mid 0 \le x \le y \le z \le 1\}$

Ans: $\{(x,y,z) \mid 0 \le z \le 1,\ 0 \le y \le z,\ 0 \le x \le y\}$.

$$\int_0^1 \int_0^z \int_0^y (48xyz)\, dx\, dy\, dz$$

inner integral: $\int_0^y 48xyz\, dx$

$= 48yz \int_0^y x\, dx$

$= 24y^3 z$

middle: $\int_0^z 24y^3 z\, dy = 24z\left(\frac{z^4 - 0}{4}\right)$

$= 6z^5$

outer: $\int_0^1 6z^5\, dz = z^6 \Big|_0^1 = 1$

$\therefore$ final answer is 1.

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Q6. the type of ans is Number

$$\int_{-3}^{3}\int_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}} \frac{(x^2+y^2)^2}{243\pi}\, dy\, dx$$

Ans: $\because x = r\cos\theta$

$y = r\sin\theta$

$x^2+y^2=r^2$

3 is the radius, (0,0) is center origins.

$$\therefore \int_0^{2\pi}\int_0^3 \frac{r^4}{243\pi}\cdot r\, dr\, d\theta$$

$$= \frac{1}{243\pi}\int_0^{2\pi}\int_0^3 r^5\, dr\, d\theta$$

$$= \frac{1}{243\pi}\int_0^{2\pi} d\theta \int_0^3 r^5\, dr$$

$$= \frac{2\pi}{243\pi}\cdot \frac{1}{6}r^6\Big|_0^3$$

$$= 1$$

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Q7. the type of ans is Number

$$\int_C \frac{4\sqrt{3}\,xyz}{3}\,ds \rightarrow \text{scalar line-integral}$$

c is the line-segment joining (0,0,0) & (1,1,1)

Ans: $(0,0,0)+[(1,1,1)-(0,0,0)]\cdot t$

$r(t) = (t,t,t)$

$r'(t) = (1,1,1)$ $\quad t = 0..1$

$\|r'(t)\| = \sqrt{3}$ $\quad ds = \|r'(t)\|\,dt$

$$\int_0^1 \frac{4\sqrt{3}\cdot t\cdot t\cdot t}{3}\cdot\sqrt{3}\,dt$$

$$= \int_0^1 4t^3\,dt$$

$= 1-0$

$= 1$

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Q8. $\int_0^3 \int_{\sqrt{\frac{y}{3}}}^{1} e^{x^3} \, dx\, dy$

the type of ans is Number

Ans: $\{(x,y) \mid 0 \le y \le 3, \sqrt{\frac{y}{3}} \le x \le 1\}$

$x = \sqrt{\frac{y}{3}}$

$x^2 = \frac{y}{3}$

$y = 3x^2$

$\therefore \{(x,y) \mid 0 \le x \le 1, 0 \le y \le 3x^2\}$

$$\int_0^1 \int_0^{3x^2} e^{x^3} \, dy\, dx$$

$$= \int_0^1 e^{x^3} \cdot 3x^2 \, dx$$

$$= e^{x^3} \Big|_0^1$$

$$= e^1 - e^0$$

$$= e - 1$$

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Q9. the type of ans is Number

$$\iiint_E \frac{5(x^2+y^2+z^2)}{4\pi}\, dV$$

$$E = \{(x,y,z) \mid x^2+y^2+z^2 \le 1\}$$

Ans: $\because x^2+y^2+z^2 = R^2$ (ρ)

$\therefore R^2 \le 1 \quad \therefore R=1 \quad 0 \le R \le 1$

$$\int_0^1 \int_0^{\pi} \int_0^{2\pi} \frac{5\rho^2}{4\pi}\, d\theta\, d\phi\, d\rho \cdot \rho^2 \cdot \sin\phi)$$

$$= \left(\int_0^1 \rho^4\, d\rho\right) \cdot \left(\int_0^{\pi} \sin\phi\, d\phi\right) \cdot \left(\int_0^{2\pi} d\theta\right) \cdot \frac{5}{4\pi}$$

$$= \frac{5}{4\pi} \cdot \left(\frac{\rho^5}{5}\Big|_0^1\right) \cdot \left(-\cos\phi\Big|_0^{\pi}\right) \cdot (2\pi - 0)$$

$$= \frac{5}{2} \cdot \frac{1}{5} \cdot (-\cos\pi + \cos 0)$$

$$= \frac{1}{2} \cdot (1+1) = 1$$

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Q10. Find $\nabla \cdot F$

$F = (\sin xy, \sin yz, \sin xz)$

the type of ans is multivarible function

$$\operatorname{div} F = \nabla \cdot F = \frac{d}{dx}(\sin xy) + \frac{d}{dy} \cdot (\sin yz) + \frac{d}{dz} \cdot (\sin xz)$$

$$= y \cdot \cos(xy) + z \cdot \cos(yz) + \cancel{z}\, x \cdot \cos(xz)$$