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Midterm Exam 2017

Q1. the type of answer is Number

Ans: Jacobian Matrix

$$\begin{vmatrix} \frac{dx}{du} & \frac{dx}{dv} & \frac{dx}{dw} \\ \frac{dy}{du} & \frac{dy}{dv} & \frac{dy}{dw} \\ \frac{dz}{du} & \frac{dz}{dv} & \frac{dz}{dw} \end{vmatrix} = \begin{vmatrix} v & u & 1 \\ w & 1 & u \\ 1 & w & v \end{vmatrix}$$

$$= v(1\cdot v - u\cdot w) + (-u)\cdot(w\cdot v - u) + 1(w^2 - 1^2)$$

$$= \cancel{v^2 - uvw - uvw + u^2 + w^2 - 1}$$

$$= \cancel{v^2 + u^2 + w^2 - 2uvw - 1}$$

$$\cancel{= 4+4+4-2\cdot(2\cdot 2\cdot 2) - 1}$$

$$\cancel{= 12 - 8}$$

$$\cancel{= 3}$$

$$= 2(2-4) - 2(4-2) + 1(4-1)$$

$$= -4 - 4 + 3$$

$$= -5$$

Q2. (i) ~~the type of ans is~~

$$\mathrm{curl}(F) = \nabla \times F$$

$$= \begin{vmatrix} i & j & k \\ \frac{d}{dx} & \frac{d}{dy} & \frac{d}{dz} \\ P & Q & R \end{vmatrix}$$

$$= i\left(\frac{d}{dy}\cdot R - Q\cdot\frac{d}{dz}\right) - j\left(R\cdot\frac{d}{dx} - P\cdot\frac{d}{dz}\right) + k\left(\frac{d}{dx}\cdot Q - P\frac{d}{dy}\right)$$

$$= i\left[(x^3 + x - \sin(x+y+z)) - (x^3 + x - \sin(x+y+z))\right]$$

$$- j\left[(3x^2y + y - \sin(x+y+z)) - (3x^2y + y - \sin(x+y+z))\right]$$

$$\cancel{+ k\left[(3x^2z + z - \sin(x+y+z)) - (3x^2z + z - \sin(x+y+z))\right]}$$

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$= (0,0,0)$

because curl(F) is (0,0,0), thus F is a conservatie vector field.

(ii) the type of ans is multivarible function

Ans: $F = \nabla f$

$f = \int 3x^2yz + yz + \cos(x+y+z)\, dx$

$= x^3yz + xyz + \sin(x+y+z) + g(y,z)$

$f_y = x^3z + xz + \cos(x+y+z) + g_y(y,z)$

$= x^3z + xz + \cos(x+y+z)$

$\therefore g_y(y,z) = 0$

g is not depend on y

$\therefore g(y,z) = h(z)$

$f_z = x^3y + xy + \cos(y+x+z) + h'(z)$

$= x^3y + xy + \cos(x+y+z)$

$\therefore h'(z) = 0$

$h(z) = C$ we can take $C=0$

$\therefore f(x,y,z) = x^3yz + xyz + \sin(x+y+z)$

(iii) the type of ans is Numbers.

$\because$ F is a conservative vector field.

$\therefore \int_C F \cdot dr = f(\sin\pi, \cos\pi+1, \sin 2\pi) - f(\sin 0, \cos 0+1, \sin 0)$

$= f(0,0,0) - f(0,2,0)$

$= -\sin 2$

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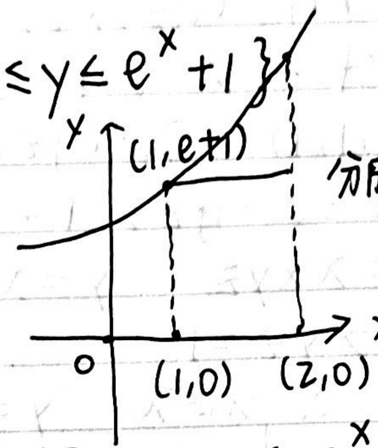
Q3. the type of ans is Sum of 2 abstract double-integrals.

$$\int_1^2 \int_0^{e^x+1} F(x,y)\,dy\,dx$$

Ans: $\{(x,y) \mid 1 \le x \le 2,\ 0 \le y \le e^x+1\}$



分成2个部分看

$y = e^x + 1$

$y - 1 = e^x$

$\ln(y-1) = x$

$x = \ln(y-1)$

type 1: $\{(x,y) \mid 1 < x < 2,\ 0 < y < e^x+1$

type 2: $\{(x,y) \mid 0 \le y \le e+1,\ 1 \le x \le 2\} \cup$

$\{(x,y) \mid e+1 \le y \le e^2+1,\ \ln(y-1) \le x \le 2.\}$.

$$\therefore \int_0^{e+1}\int_1^2 F(x,y)\,dx\,dy + \int_{e+1}^{e^2+1}\int_{\ln(y-1)}^{2} F(x,y)\,dx\,dy$$

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Q4. the type of ans is Numbers

$f = x+y+z \qquad xyz = 1$

Ans: $\nabla f = (1,1,1) \qquad g = xyz - 1$

$\nabla g = (yz, xz, xy)$

$\nabla f = \lambda \cdot \nabla g$

$(1,1,1) = \lambda \cdot (yz, xz, xy)$

$1 = \lambda \cdot yz, \quad 1 = \lambda \cdot xz, \quad 1 = \lambda \cdot xy$

$x = \lambda xyz \qquad y = \lambda \cdot xyz \qquad z = \lambda \cdot xyz$

$\because xyz = 1$

$\therefore x = \lambda \quad y = \lambda \quad z = \lambda$

$x \cdot y \cdot z = \lambda^3$

$1 = \lambda^3$

$\lambda = 1$

$\therefore$ the location is $(1,1,1)$

~~fat~~ $f = x+y+z$

$= 1+1+1 = 3$

$\therefore$ the smallest value is 3.

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Q5. the type of ans is Number

$$\iiint_E 48xyz\, dV$$

$$\{(x,y,z) \mid 0 \le x \le y \le z \le 1\}$$

Ans: $\{(x,y,z) \mid 0 \le z \le 1,\ 0 \le y \le z,\ 0 \le x \le y\}$.

$$\int_0^1 \int_0^z \int_0^y (48xyz)\, dx\, dy\, dz$$

inner integral: $\int_0^y 48xyz\, dx$

$$= 48yz \int_0^y x\, dx$$

$$= 24y^3 z$$

middle: $\int_0^z 24y^3 z\, dy = 24z\left(\frac{z^4 - 0}{4}\right)$

$$= 6z^5$$

outer: $\int_0^1 6z^5\, dz = z^6 \Big|_0^1 = 1$

$\therefore$ final answer is 1.

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Q6. the type of ans is Number

$$\int_{-3}^{3}\int_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}} \frac{(x^2+y^2)^2}{243\pi}\,dy\,dx$$

Ans: $\because x = r\cos\theta$

$y = r\sin\theta$

$x^2+y^2=r^2$

3 is the radius, (0,0) is center origins.

$$\therefore \int_0^{2\pi}\int_0^3 \frac{r^4}{243\pi}\cdot r\,dr\,d\theta$$

$$= \frac{1}{243\pi}\int_0^{2\pi}\int_0^3 r^5\,dr\,d\theta$$

$$= \frac{1}{243\pi}\int_0^{2\pi} d\theta \int_0^3 r^5\,dr$$

$$= \frac{2\pi}{243\pi}\cdot \frac{1}{6}r^6\Big|_0^3$$

$$= 1$$

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Q7. the type of ans is Number

$$\int_C \frac{4\sqrt{3}\, xyz}{3}\, ds \rightarrow \text{scalar line-integral}$$

c is the line-segment joining (0,0,0) & (1,1,1)

Ans: $(0,0,0)+[(1,1,1)-(0,0,0)]\cdot t$

$r(t) = (t, t, t)$

$r'(t) = (1,1,1)$ $\quad t = 0..1$

$\|r'(t)\| = \sqrt{3}$ $\quad ds = \|r'(t)\|\, dt$

$$\int_0^1 \frac{4\sqrt{3}\cdot t\cdot t\cdot t}{3}\cdot\sqrt{3}\, dt$$

$$= \int_0^1 4t^3\, dt$$

$= 1-0$

$= 1$

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Q8. $\int_0^3 \int_{\sqrt{\frac{y}{3}}}^1 e^{x^3} \, dx\, dy$

the type of ans is Number

Ans: $\{(x,y) \mid 0 \le y \le 3, \sqrt{\frac{y}{3}} \le x \le 1\}$

$x = \sqrt{\frac{y}{3}}$

$x^2 = \frac{y}{3}$

$y = 3x^2$

$\therefore \{(x,y) \mid 0 \le x \le 1, 0 \le y \le 3x^2\}$

$\int_0^1 \int_0^{3x^2} e^{x^3} \, dy\, dx$

$= \int_0^1 e^{x^3} \cdot 3x^2 \, dx$

$= e^{x^3} \Big|_0^1$

$= e^1 - e^0$

$= e - 1$

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Q9. the type of ans is Number

$$\iiint_E \frac{5(x^2+y^2+z^2)}{4\pi}\, dV$$

$$E = \{(x,y,z) \mid x^2+y^2+z^2 \le 1\}$$

Ans: $\because x^2+y^2+z^2 = R^2$ (ρ)

$\therefore R^2 \le 1$ $\therefore R = 1$ $0 \le R \le 1$

$$\int_0^1 \int_0^\pi \int_0^{2\pi} \frac{5\rho^2}{4\pi}\, d\theta\, d\phi\, d\rho \cdot \rho^2 \cdot \sin\phi)$$

$$= \left(\int_0^1 \rho^4\, d\rho\right) \cdot \left(\int_0^\pi \sin\phi\, d\phi\right) \cdot \left(\int_0^{2\pi} d\theta\right) \cdot \frac{5}{4\pi}$$

$$= \frac{5}{4\pi} \cdot \left(\frac{\rho^5}{5}\Big|_0^1\right) \cdot \left(-\cos\phi\Big|_0^\pi\right) \cdot (2\pi - 0)$$

$$= \frac{5}{2} \cdot \frac{1}{5} \cdot (-\cos\pi + \cos 0)$$

$$= \frac{1}{2} \cdot (1+1) = 1$$

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Q10. Find $\nabla \cdot F$

$F = (\sin xy, \sin yz, \sin xz)$

the type of ans is multivarible function

$\operatorname{div} F = \nabla \cdot F = \frac{d}{dx}(\sin xy) + \frac{d}{dy} \cdot (\sin yz) + \frac{d}{dz} \cdot (\sin xz)$

$= y \cdot \cos(xy) + z \cdot \cos(yz) + x \cdot \cos(xz)$

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Simial Question as 2017 Midterm Exam

Q1. Find the Jacobian of the transformation from (r,t) to (x,y)

$$G(r,t) = (r\sin t\ ,\ t-\cos t)$$
$$(r,t) = (1,\pi)$$

Ans: $\begin{vmatrix} \frac{dx}{dr} & \frac{dx}{dt} \\ \frac{dy}{dr} & \frac{dy}{dt} \end{vmatrix} = \frac{dx}{dr}\cdot\frac{dy}{dt} - \frac{dy}{dr}\cdot\frac{dx}{dt}$

$$= \sin t\cdot(1+\sin t) - 0\cdot r\cos t$$
$$= \sin\pi\cdot(1+\sin\pi)$$
$$= 0.$$

the Jacobian of G is 0.

Q2. Show that

(i) $F = (y^2,\ 2xy+e^z,\ ye^z)$ is a conservative function

Ans: curl (F) is $\nabla\times F$

$$= \begin{vmatrix} i & j & k \\ \frac{d}{dx} & \frac{d}{dy} & \frac{d}{dz} \\ y^2 & 2xy+e^z & ye^z \end{vmatrix}$$

$$= (e^z - e^z)i - (0-0)j + k(2y\cancel{0} - 2y)$$

$$= (0,0,0)$$

∴ the F is conservative vector field.

(ii) Find a function $f(x,y,z)$ such that $F=\nabla f$

Ans: $f = \int y^2\,dx = xy^2 + g(y,z)$

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$\frac{d}{dy} \cdot f = 2xy + g_y(y,z) = 2xy + e^z$

$g_y(y,z) = e^z$

$g(y,z) = e^z \cdot y + h(z)$

$f = xy^2 + y \cdot e^z + h(z)$

$\frac{d}{dz} \cdot f = 0 + y \cdot e^z + h'(z) = ye^z$

$\therefore h'(z) = 0$

$h(z) = C$ we can take C as 0.

$\therefore f = xy^2 + ye^z$

(iii) Find the line integral $\int_C F \cdot dr$ where C is a curve.

$r = (5t, \sin t - 1, 2\pi \cdot t) \quad 0 \le t \le \pi.$

Ans: $\int_C F \cdot dr = f(5\pi, \sin\pi - 1, 2\pi \cdot \pi)$

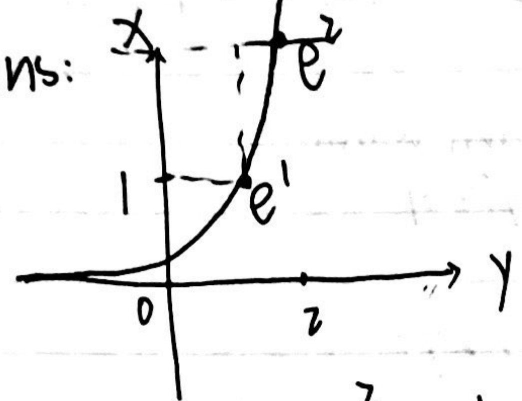
$- f(0, -1, 0)$

≈ -373791517.5

Q3. Sketch the region of integration and change the order of integration.

$$\int_0^2 \int_1^{e^y} F(x,y)\, dx\, dy$$

Ans:



$$\begin{cases} 0 \le y \le 2 \\ 1 \le x \le e^y \end{cases}$$

$$x = e^y$$

$$\ln x = y$$

$$y = \ln x.$$

$$\therefore \int_1^{e^2} \int_0^{e^1} F(x,y)\, dy\, dx + \int_1^{\ln x} \int_{e^1}^{e^2} F(x,y)\, dy$$

Q4. use largange to find the mini value of

$f(x,y) = 2x+3y \quad x^2+y^2=4$

Ans:

$\therefore \quad \nabla f = (2,3) \qquad \nabla g = (2x, 2y)$

$\nabla f = \lambda \nabla g \qquad 2x\lambda = 2 \qquad 2y\lambda = 3$

$(2,3) = (2x, 2y)\lambda \qquad \lambda = \frac{1}{x} \qquad \lambda = \frac{3}{2}y$

$x = \frac{1}{\lambda} \qquad y = \frac{3}{2}\lambda$

$x^2+y^2=4$

$\left(\frac{1}{\lambda}\right)^2 + \left(\frac{3}{2}\lambda\right)^2 = 4$

$\lambda = \pm 1.2152, \pm 0.548$

$\lambda = 1.2152 \quad x = 0.8229 \quad y = 1.82265 \qquad f = 7.11375.$

$\lambda = -1.2152 \quad x = -0.8229 \quad y = -1.82265 \qquad f = -7.11375$

$\lambda = -0.548 \quad x = -1.824 \quad y = -0.822 \qquad f = -6.114.$

$\therefore$ when $\lambda = -1.2152$, f minimum value is -7.11375.

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Q5. compute the volume integral

$$\iiint_E 3xy^2z\,dv$$

$$\{(x,y,z) \mid 0 \le x \le y \le z \le 1\}.$$

Ans: $\{(x,y,z) \mid 0 \le z \le 1,\ 0 \le y \le z,\ 0 \le x \le y\}.$

$$\because \int_0^1 \int_0^z \int_0^y 3xy^2z\,dx\,dy\,dz$$

$$\cancel{= \int_0^1 z\,dz \cdot \int_0^z y^2\,dy \cdot \int_0^y 3x\,dx}$$

$$= \cancel{\frac{z^2}{2}\Big|_0^1 \cdot \frac{y^3}{3}\Big|_0^z.}$$

$$\int_0^y 3xy^2z\,dx = \frac{3}{2}x^2y^2z\Big|_0^y = \frac{3}{2}y^4z$$

$$\int_0^z \frac{3}{2}y^4z\,dy = \frac{3}{10}y^5z\Big|_0^z = \frac{3}{10}z^6$$

$$\int_0^1 z^6 \cdot \frac{3}{10}\,dz = \frac{3}{70}z^7\Big|_0^1 = \frac{3}{70}$$

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Q6. By converting to polar coordinates, compute.

$$\int_{-4}^{4}\int_{\sqrt{8-x^2}}^{\sqrt{8-x^2}} \frac{(x^2+y^2)}{2}\,dy\,dx$$

Ans: $\{0 \le r \le 4 \quad 0 \le \theta \le 2\pi\}$. $x^2+y^2=r^2$.

$$\therefore \int_0^{2\pi}\int_0^4 \frac{r^2}{2}\,dr\,d\theta$$

$$= \int_0^{2\pi} \frac{r^3}{6}\Big|_0^4\,d\theta$$

$$= \frac{64}{3}\pi.$$

Q7. compute the line integral.

$$\int_C \frac{x^2yz\cdot\pi}{2}\,ds.$$

C is a line segment joining $(0,1,1)$ to $(1,2,0)$

$(0, \cancel{a}\, 1, 1) + [(1,2,0)-(0,1,1)]t$

$= (0,1,1) + (1,1,-1)t = (t, 1+t, 1-t) \quad 0 \le t \le 1.$

$x=t, \ y=1+t, \ z=1-t$

$r(t) = (t, 1+t, 1-t) \quad r'(t) = (1,1,-1)$

$\|r'(t)\| = \sqrt{3}$

$$\int_0^1 \frac{t^2\cdot(1+t)\cdot(1-t)\cdot\pi}{2}\,dt\cdot\sqrt{3} = \frac{\sqrt{3}}{15}\cdot\pi.$$

Q8. compute $\int_0^1 \int_0^{\sqrt{1-x^2}} e^{y^2}\, dy\, dx$

$\Rightarrow \{(x,y) \mid 0 \le y \le \sqrt{1-x^2},\ 0 \le x \le 1\}.$

$y = \sqrt{1-x^2}$

$x = \cancel{y^2-1}\ \sqrt{1-y^2}$

$\{(x,y) \mid \sqrt{1-y^2} \le x \le 1,\ 0 \le y \le 1\}.$

$$\int_0^1 \int_{\sqrt{1-y^2}}^1 e^{y^2}\, dx\, dy.$$

$= 0.4194.$

Q9. compute the volume integral

$$\iiint_E x^2\, dV.$$

$x^2+y^2=4$

$z=0$

$z^2=2(x^2+y^2)$

Ans: $z^2 = 2(x^2+y^2)$

$= 2r^2$

$z = \sqrt{2r^2}$

$= \sqrt{2}\, r$

$x^2+y^2 = r^2 = 4$

$r = 2$

$\begin{cases} 0 \le r \le 2 \\ 0 \le \theta \le 2\pi \end{cases}$

$0 \le z \le \sqrt{2}\, r$

$$\int_0^{2\pi} \int_0^2 \int_0^{\sqrt{2}r} x^2 \cdot r\, dz\, dr\, d\theta$$

$$= \frac{32\sqrt{2} \cdot \left(\frac{\cos 2\pi \sin 2\pi}{2} + \pi\right)}{5}.$$

(the question is complex, haha

Q10. find $\nabla \cdot F$ if $F = (2\sin(x,y),\ \cos(y,z),\ 3\sin(\overset{xz}{\cancel{x,y}}))$.

Ans: $\nabla \cdot F = \frac{d}{dx}(2\sin(xy)) + \frac{d}{dy}(\cos(yz)) + \frac{d}{dz}(3\sin(xz))$

$= 2y\cos(xy) + (-z\sin(yz)) + 3x\cos(xz)$