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Find Answer(s)

1, -5.

2, ① $f(x,y,z) = (x^3yz + xyz + \sin(x+y+z))\,i + (x^3zy + xzy + \sin(x+y+z))\,j + (x^3yz + xyz + \sin(x+y+z))\,k$. $x^3yz + xyz + \sin(x+y+z)$

③ $\langle -\sin z, -\sin z, -\sin z \rangle$. $-\sin z$

3, $\int_{e+1}^{e^2+1} \int_{\ln(y-1)}^{2} F(x,y)\,dx\,dy$.

4, 3.

5, 1.

6, 1.

7, $\frac{\sqrt{3}}{3}$ 1

8, $\int_0^1 \int_{3x^2}^{3} e^{x^3}\,dy\,dx = -\frac{7}{9} + \frac{4}{9}e^3$. $e-1$

9, 1

10, $\langle -y\cos(yz), -z\cos(xz), -x\cos(xy) \rangle$

$\langle y\cos(yz), z\cos(xz), x\cos(xy) \rangle$.

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1. Find the Jacobian of the transformation from (u,v,w)-space to (x,y,z)-space.

$$x = uv + w \qquad y = uw + v \qquad z = vw + u$$

at the point $(u,v,w) = (2,2,2)$.

$$\begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} & \frac{\partial x}{\partial w} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} & \frac{\partial y}{\partial w} \\ \frac{\partial z}{\partial u} & \frac{\partial z}{\partial v} & \frac{\partial z}{\partial w} \end{vmatrix}$$

$$\frac{\partial x}{\partial u} = v \quad \frac{\partial x}{\partial v} = u \quad \frac{\partial x}{\partial w} = 1.$$

$$\frac{\partial y}{\partial u} = w \quad \frac{\partial y}{\partial v} = 1 \quad \frac{\partial y}{\partial w} = u$$

$$\frac{\partial z}{\partial u} = 1 \quad \frac{\partial z}{\partial v} = w \quad \frac{\partial z}{\partial w} = v.$$

$$\begin{vmatrix} v & u & 1 \\ w & 1 & u \\ 1 & w & v \end{vmatrix} = v(v-uw) - u(wv-u) + 1(w^2-1)$$

$$= v^2 - uwv - uwv + u^2 + w^2 - 1$$

$$= v^2 + u^2 + w^2 - 2uwv - 1.$$

plug in $(u,v,w) = (2,2,2)$

~~Ans~~: $v^2 + u^2 + w^2 - 2uwv - 1.$

$= 4 + 4 + 4 - 16 - 1$

$= 12 - 17$

$= -5.$

Ans: -5.

(i) Show that

$F = \langle 3x^2yz + yz + \cos(x+y+z),\ x^3z + xz + \cos(x+y+z),$
$x^3y + xy + \cos(x+y+z) \rangle.$

is a conservative vector field.

(ii) Find a function $f(x,y,z)$ such that $F = \nabla f$.

(iii) Find the line-integral $\int_C F \cdot dr$ where C is the curve

$r = \langle \sin t, \cos t + 1, \sin 2t \rangle, \quad 0 \le t \le \pi$.

(i)
$$\begin{matrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ P & Q & R \end{matrix}$$

$i: R\_y = x^3 + x - \sin(x+y+z)$

$Q\_z = x^3 + x - \sin(x+y+z)$

$R\_y = Q\_z$

$j: R\_x = 3x^2y + y - \sin(x+y+z)$

$P\_z = 3x^2y + y - \sin(x+y+z)$

$R\_x = P\_z$

$k: Q\_x = 3x^2z + z - \sin(x+y+z)$

$P\_y = 3x^2z + z - \sin(x+y+z)$

$Q\_x = P\_y$

F is a conservative vector field because $\text{curl}(F) = \langle 0,0,0 \rangle$.

(ii) $f(x,y,z) = (x^3yz + xyz + \sin(x+y+z))i + (x^3zy + xzy + \sin(x+y+z))j + (x^3yz + xyz + \sin(x+y+z))k$. $= x^3yz + xyz + \sin(x+y+z)$

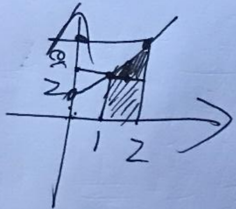
(iii) $f(0,0,0) - f(0,2,0) = \langle 0,0,0 \rangle - \langle \sin 2, \sin 2, \sin 2 \rangle$
$= \langle -\sin 2, -\sin 2, -\sin 2 \rangle. = -\sin 2.$

3, Sketch the region of integration and change the order of integration

$$\int_1^2 \int_0^{e^x+1} F(x,y)\,dy\,dx.$$

Ans:

$$\cancel{\int_0^{e^2+1} \int_1^{\ln(y-1)} F(x,y)\,dx\,dy}$$



$y = e^x + 1$

$e^x = y - 1$

$x = \ln(y-1)$

$$\int_{e+1}^{e^2+1} \int_{\ln(y-1)}^{2} F(x,y)\,dx\,dy$$

4. Use Largange multipliers (no credit for other methods) to find the smallest value that $x+y+z$ can be, given that $xyz=1$.

$\nabla f = \langle 1, 1, 1 \rangle$

$\nabla g = \langle yz, xz, xy \rangle$

$yz = \lambda \quad xz = \lambda \quad xy = \lambda$

$(xyz)^2 = \lambda^3 = 1$

$\lambda^3 = 1$

$\lambda = 1$

$\boxed{f(x,y,z) = 3.}$

6. compute the volume integral

$$\iiint_E 48xyz\, dV$$

where E is the region in 3D.

$$\{(x,y,z) \mid 0 \le x \le y \le z \le 1\}$$

$$\int_0^1 \int_x^1 \int_y^1 48xyz\, dz\, dy\, dx$$

$= 1.$

6. By converting to polar coordinate, compute

$$\int_{-3}^{3}\int_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}} \frac{(x^2+y^2)^2}{243\pi}\,dy\,dx$$

$x = r\cos\theta \quad y = r\sin\theta$

$D = \{(r,\theta),\ 0 \le \theta \le 2\pi,\ 0 \le r \le 3\}$.

$$\int_0^3\int_0^{2\pi} \frac{r^5}{243\pi}\,d\theta\,dr$$

$= 1$

7. compute the line integral

$$\int_C \frac{4\sqrt{3}xyz}{3}\,ds$$

where C is the line-segment joining (0,0,0) and (1,1,1)

line: A + t(B-A) = (t,t,t) t=0..1.

$r(t) = \langle t,t,t \rangle$

$r'(t) = \langle 1,1,1 \rangle$

$|r'(t)| = \sqrt{3}$

$$\cancel{\int_0^1 \frac{4\sqrt{3}t^3}{3}\,dt}$$

$$\int_0^1 \frac{4\sqrt{3}t^3}{3}\,dt.$$

$$\cancel{= \frac{\sqrt{3}}{3}}$$

$$\int_0^1 \frac{4\sqrt{3}t^3}{3}\sqrt{3}\,dt = \int_0^1 4t^3\,dt$$

$$= \frac{4t^4}{4}\Big|_0^1$$

$$= t^4\Big|_0^1$$

$$= 1-0$$

$$= 1.$$

9, (

8, Compute

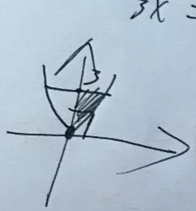
$$\int_0^3 \int_{\sqrt{\frac{y}{3}}}^1 e^{x^3}\,dx\,dy$$

wh (Hint: Not even Dr. Z can do $\int e^{x^3}dx$, so you must be clever and first change the order of integration.)

$x = \sqrt{\frac{y}{3}}$

$x^2 = \frac{y}{3}$

$3x^2 = y$

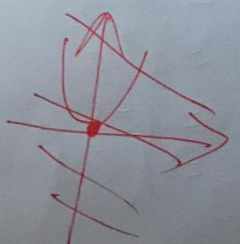


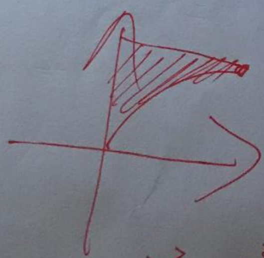
$$\int_0^1 \int_{3x^2}^3 e^{x^3}\,dy\,dx.$$

$$= \int_0^1 e^{x^3}(3-3x^2)\,dx$$

$$= -\frac{7}{9} + \frac{4e^3}{4}$$

=





$$\int_0^1 \int_0^{3x^2} e^{x^3}\,dy\,dx$$

inner: $\int_0^{3x^2} e^{x^3} dy = e^{x^3}\int_0^{3x^2} dy$

$= 3x^2 e^{x^3}$

outer: $\int_0^1 3x^2 e^{x^3} dx$

$u = x^3$

$du = 3x^2 dx$

$\int_0^1 e^u du$

$= e^u = e^1 - e^0 = e-1.$

$\boxed{\text{Ans: } e-1}$

9. Compute the volume integral

$$\iiint_E \frac{5(x^2+y^2+z^2)}{4\pi}\, dV$$

where

$$E = \{(x,y,z) \mid x^2+y^2+z^2 \le 1\}.$$

$x = \rho\sin\phi\cos\theta \quad y = \rho\sin\phi\sin\theta$

$z = \rho\cos\phi \quad dV = \rho^2 \sin\phi\, d\rho\, d\theta\, d\phi$

$\{(r,\theta,\phi) \mid 0 \le r \le 1,\ 0 \le \theta \le 2\pi,\ 0 \le \phi \le \pi\}.$

$$\int_0^1 \int_0^{2\pi} \int_0^{\pi} \frac{5\left((\rho\sin\phi\cos\theta)^2 + (\rho\sin\phi\sin\theta)^2 + (\rho\cos\phi)^2\right)}{4\pi}\, \rho^2 \sin\phi\, d\phi\, d\theta\, d\rho$$

$= 1.$

10. Find $\nabla \cdot F$ if

$$F = \langle \sin(xy), \sin(yz), \sin(xz) \rangle$$

$$\begin{vmatrix} i & j & k \\ \frac{d}{dx} & \frac{d}{dy} & \frac{d}{dz} \\ \sin(xy) & \sin(yz) & \sin(xz) \end{vmatrix}$$

$$i(0 - (\cos yz)y) - j(z(\cos xz) - 0)$$

$$+k(0 - x\cos(xy))$$

$$= \langle -y\cos(yz), -z\cancel{c}os(xz), -x\cos(xy) \rangle.$$

Ans:

$$\langle y\cos(yz), \; z\cos(xz), \; x\cos(xy) \rangle$$

$$\nabla \cdot F = \operatorname{div} F = \frac{d}{dx} P + \frac{d}{dy} Q + \frac{d}{dz} R.$$