"QUIZ" for Lecture 20

NAME: (print!) Gillian Mulvoy **Section:** ____

E-MAIL SCANNED .pdf OF COMPLETED QUIZ to DrZcalc3@gmail.com (Attachment: q20FirstLast.pdf) ASAP BUT NO LATER THAN Nov. 16, 8:00pm

1. Find an equation for the tangent plane to the parametric surface

$$x = v^2 \ , \quad y = u + v \ , \quad z = u^2 \ ,$$

$1 = v^2$ $\quad 2 = v + u$ $\quad 1 = u^2$ $\quad u = 1 \quad v = 1$

at the point $(1, 2, 1)$. Simplify as much as you can!

$\langle v^2, u+v, u^2 \rangle$

$r_v = \langle 2v, 1, 0 \rangle \qquad r_u = \langle 0, 1; 2u \rangle$

$r_v = \langle 2, 1, 0 \rangle \qquad r_u = \langle 0, 1, 2 \rangle$

$$\begin{vmatrix} i & j & k \\ 2 & 1 & 0 \\ 0 & 1 & 2 \end{vmatrix} = 2i - 4j + 2k = \langle 2, -4, 2 \rangle$$

$2(x-1) + 4(y-2) + 2(z-1) = 0$

$2x - 4y + 2z = -4 \quad \to x - 2y + z = -2$

2. Evaluate the surface integral

$$\iint_S z \, dS \ ,$$

where S is the triangular region with vertices $(2, 0, 0)$, $(0, 2, 0)$, $(0, 0, 2)$.

$z = 1 - x - y$

$x + y = -1 \qquad x + y = 1$

$\sqrt{3} \int_0^{2\pi} \int_0^2 r \cdot r \, dr \, d\theta$

$\frac{r^3}{3} \Big|_0^2 = \frac{8}{3}$

$\int_0^{2\pi} \frac{8\sqrt{3}}{3} \, d\theta = \frac{8\sqrt{3}\pi}{3}$

"QUIZ" for Lecture 22

NAME: (print!) Gillian Mulvoy **Section:** ______

E-MAIL SCANNED .pdf OF COMPLETED QUIZ to DrZcalc3@gmail.com (Attachment: q22FirstLast.pdf) ASAP BUT NO LATER THAN Nov. 16, 8:00pm

Evaluate the surface integral $\int\int_S \mathbf{F} \cdot d\mathbf{S}$ for the given vector field $\mathbf{F}$ and oriented surface S.

$$\mathbf{F}(x, y, z) = \langle xy \, , \, yz \, , \, zx \rangle \quad ,$$

and S is the part of the paraboloid $z = 1 - x^2 - y^2$ that lies above the square $0 \leq x \leq 1 \, , \, 0 \leq y \leq 1$ and has upward orientation.

$g = 1 - x^2 - y^2 \qquad g_x = -2x \qquad g_y = -2y$

$P = xy \qquad Q = yz \qquad R = zx$

$$\iint_D \left(-P \frac{\partial g}{\partial x} - Q \frac{\partial g}{\partial y} + R\right) dA$$

$$\iint_D (-xy \cdot -2x - yz \cdot -2y + zx) \, dA$$

$$\iint_D (2x^2y + 2y^2z + zx) \, dA$$

$$\iint_D (2x^2y + (2y^2 + x)(1 - x^2 - y^2)) \, dA$$

$$\int_0^1 \int_0^1 (2x^2y + 2y^2 - 2y^2x^2 - 2y^4 + x - x^3 - xy^2) \, dx \, dy$$

$$\int_0^1 \frac{2x^3}{3}y + 2xy^2 - 2y^2\frac{x^3}{3} - 2xy^4 + \frac{x^2}{2} - \frac{x^4}{4} - \frac{x^2}{2}y^2 \Big|_0^1 \, dy$$

$$\int_0^1 \frac{2y}{3} + 2y^2 - \frac{2y^2}{3} - 2y^4 + \frac{1}{4} - \frac{y^2}{2} \, dy$$

$$\frac{y^2}{3} + y^3 - \frac{2y^3}{9} - \frac{2y^5}{5} + \frac{y}{4} - \frac{y^3}{6} \Big|_0^1 = \frac{\cancel{1}^{3}}{\cancel{3}_{9}} + \frac{\cancel{1}^{9}}{9} - \frac{2}{9} - \frac{2}{5} + \frac{1}{4} - \frac{1}{6}$$

$$= \frac{4}{3} - \frac{2}{5} + \frac{1}{4} - \frac{1}{6} = \frac{61}{60}$$