

"QUIZ" for Lecture 20

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E-MAIL SCANNED .pdf OF COMPLETED QUIZ to DrZcalc3@gmail.com (Attachment: q20FirstLast.pdf) ASAP BUT NO LATER THAN Nov. 16, 8:00pm

1. Find an equation for the tangent plane to the parametric surface

$$x = v^2, \quad y = u + v, \quad z = u^2,$$

at the point (1, 2, 1). Simplify as much as you can!

$$\begin{array}{l} r = v^2 i + (u+v)j + u^2 k \\ r_u = j + 2uk \\ r_v = 2vi + j \end{array} \quad \begin{array}{l} 1 = v^2 \quad v = 1 \\ 2 = u + v \quad u = 1 \\ 1 = u^2 \end{array} \quad \left. \begin{array}{l} -2x + 2 + 4y - 8 - 2z + 2 = 0 \\ -2x + 4y - 2z = 4 \end{array} \right\}$$

$$r_u \times r_v = \begin{vmatrix} i & j & k \\ 0 & 1 & 2u \\ 2v & 1 & 0 \end{vmatrix} = (0 - 2u)i - (0 - 4uv)j + (0 - 2v)k$$

$$= -2ui + 4uvj - 2vk$$

@(1, 2, 1) = -2i + 4j - 2k

$$-2(x-1) + 4(y-2) - 2(z-1) = 0$$

2. Evaluate the surface integral

$$\iint_S z \, dS,$$

where S is the triangular region with vertices (2, 0, 0), (0, 2, 0), (0, 0, 2).

$$\begin{aligned} \frac{x}{2} + \frac{y}{2} + \frac{z}{2} &= 1 \\ x + y + z &= 2 \\ z &= 2 - x - y \end{aligned}$$

$$\iint_S z \, dS = \iint_D z \sqrt{(-1)^2 + (-1)^2 + 1} \, dA$$

$$= \sqrt{3} \iint_D z \, dA$$

$$\{(x, y) \in D \mid 0 \leq x \leq 2, 0 \leq y \leq -x + 2\}$$

$$\sqrt{3} \int_0^2 \int_0^{-x+2} z \, dz \, dx = \sqrt{3} \left[\frac{z^2}{2} \right]_0^{-x+2} = \frac{(-x+2)^2}{2} \sqrt{3}$$

$$\frac{\sqrt{3}}{2} \int_0^2 (-x+2)^2 \, dx = \frac{\sqrt{3}}{2} \left[\frac{x^3}{3} - 2x^2 + 4x \right]_0^2 = \frac{4\sqrt{3}}{3}$$