q20 Rahul Paleja

Section 22

(1) $x=v^2$, $y=u+v$, $z=u^2$
at the point $(1,2,1)$

$1=v^2$ $\quad 2=u+v$ $\quad 1=u^2$ $\quad u=1, v=1$

$r=v^2 i+(u+v) j+(u^2)K$

$r_u=0i+1j+2k$ $\quad r_v=2vi+1j+0K$

Plug in $u=1$ & $v=1$

$r_u=\langle 0,1,2\rangle$ $\quad r_v=\langle 2,1,0\rangle$

$$r_u \times r_v = \begin{vmatrix} i & j & K \\ 0 & 1 & 2 \\ 2 & 1 & 0 \end{vmatrix} \Rightarrow i(-2)-j(-4)+K(-2)$$

$=-2i+4j-2K$

$\langle -2,4,-2\rangle$

Use Point: $(1,2,1)$

$-2(x-1)+4(y-2)-2(z-1)=0$

$-2x+2+4y-8-2z+2=0$

$\dfrac{-2x+4y-2z}{-2}=\dfrac{4}{-2} \Rightarrow \boxed{x-2y+z=-2}$

(2) $\iint_S z\,dS$ where S is triangular region with vertices $(2,0,0),(0,2,0),(0,0,2)$

$\frac{x}{2}+\frac{y}{2}+\frac{z}{2}=1$ $\quad 2(\frac{z}{2}=1-\frac{y}{2}-\frac{x}{2})$

$z=2-y-x$

$\langle x,y,2-y-x\rangle$

$r=\langle u,v,2-v-u\rangle$ $u+v$ are parameters

$r_u=\langle 1,0,-1\rangle$ $\quad r_v=\langle 0,1,-1\rangle$

$$r_u \times r_v = \begin{vmatrix} i & j & k \\ 1 & 0 & -1 \\ 0 & 1 & -1 \end{vmatrix} = i(1)-j(-1)+k(1)=i+j+k=\langle 1,1,1\rangle$$

$ds=\sqrt{3}\,du\,dv$

$\int_0^2\int_0^{2-u} 2-v-u\sqrt{3}\,du\,dv$