"QUIZ" for Lecture 20

NAME: (print!) Prathik Colla **Section:** _____

E-MAIL SCANNED .pdf OF COMPLETED QUIZ to DrZcalc3@gmail.com (Attachment: q20FirstLast.pdf) ASAP BUT NO LATER THAN Nov. 16, 8:00pm

1. Find an equation for the tangent plane to the parametric surface

$$x = v^2 \quad , \quad y = u + v \quad , \quad z = u^2 \quad , \quad \begin{matrix} u=1 \\ v=1 \end{matrix}$$

$1 = v^2$ $\quad 2 = 1+1$ $\quad 1 = u^2$

at the point $(1, 2, 1)$. Simplify as much as you can!

$r = xi + yj + zk \Rightarrow r = v^2 i + (u+v)j + u^2 k$

$r_u = x_u i + y_u j + z_u k \Rightarrow r_u = j + 2uk$

$r_v = x_v i + y_v j + z_v k \Rightarrow r_v = 2vi + j$

Plug in $(u,v) \to (1,1)$

$r_u(1,1) = j + 2k$, $r_v = 2i + j$

$$r_u \times r_v = \begin{vmatrix} i & j & k \\ 0 & 1 & 2 \\ 2 & 1 & 0 \end{vmatrix} = \langle -2, 4, -2 \rangle$$

$(x_0, y_0, z_0) = (1, 2, 1)$

$a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$

$-2(x-1) + 4(y-2) - 2(z-1) = 0$

$-2x + 2 + 4y - 8 - 2z + 2 = 0$

$-2x + 4y - 2z = -2 + 8 - 2$

$-2x + 4y - 2z = 4$

$-2(x - 2y + z) = 4$

$$\boxed{x - 2y + z = -2}$$

2. Evaluate the surface integral

$$\int\int_S z \, dS \quad ,$$

where S is the triangular region with vertices $(2, 0, 0), (0, 2, 0), (0, 0, 2)$.

$x + y + z = 2$

$z = 2 - x - y$

$\iint_S f(x,y,z)\, dS = \iint_D f(x, y, g(x,y)) \sqrt{1 + g_x^2 + g_y^2}\, dx\, dy$

$g(x,y) = 2 - x - y \Rightarrow g_x = -1, \; g_y = -1$

$\iint_S z \, dS = \iint_D (2 - x - y)\sqrt{3}\, dy\, dx$

$D = \{(x,y) \mid x \geq 0, y \geq 0, x + y \leq 2\}$

$D = \{(x,y) \mid 0 \leq x \leq 2, 0 \leq y \leq 2 - x\}$

$\int_0^2 \int_0^{2-x} \sqrt{3}\, dy\, dx = 2\sqrt{3}$

$$\boxed{2\sqrt{3}}$$