"QUIZ" for Lecture 20

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E-MAIL SCANNED .pdf OF COMPLETED QUIZ to DrZcalc3@gmail.com (Attachment: q20FirstLast.pdf) ASAP BUT NO LATER THAN Nov. 16, 8:00pm

1. Find an equation for the tangent plane to the parametric surface

$$x = v^2 \quad , \quad y = u + v \quad , \quad z = u^2 \quad ,$$

at the point $(1, 2, 1)$. Simplify as much as you can!

$u^2=1 \quad v^2=1$
$u=1 \quad v=1$

$r = u^2 i + (u+v) j + v^2 k$

$r_u = 2ui + j \quad r_u(1,1) = \langle 2,1,0 \rangle$

$r_v = j + 2vk \quad r_v(1,1) = \langle 0,1,2 \rangle$

$\langle 2,1,0 \rangle \times \langle 0,1,2 \rangle$

$$\begin{vmatrix} i & j & k \\ 2 & 1 & 0 \\ 0 & 1 & 2 \end{vmatrix} = (2-0)i - (4-0)j + (2-0)k$$

$= \langle 2,-4,2 \rangle$

$= \langle 1,-2,1 \rangle$

$1(x-1) - 2(y-2) + 1(z-1) = 0$

$x - 1 - 2y + 4 + z - 1 = 0$

$x - 2y + z = -2$

2. Evaluate the surface integral

$$\int\!\!\int_S z \, dS \quad ,$$

where S is the triangular region with vertices $(2,0,0), (0,2,0), (0,0,2)$.
P Q R

$PQ = \langle -2,2,0 \rangle$
$\times$
$PR = \langle -2,0,2 \rangle$

$$= \begin{vmatrix} i & j & k \\ -2 & 2 & 0 \\ -2 & 0 & 2 \end{vmatrix} = (4-0)i - (-4-0)j + (0+4)k$$

$= \langle 4,4,4 \rangle = \langle 1,1,1 \rangle$

$1(x-2) + 1(y) + 1(z) = 0$

$x + y + z = 2$

$ds = \sqrt{1 + (-1)^2 + (-1)^2} = \sqrt{3}$

$\{(r,\theta) : 0 \le r \le 1 , 0 \le \theta \le 2\pi\}$

$z = 2 - x - y = 0$
$x + y = 2$

$$\iint 2 - x - y \, dS = \int_0^{2\pi}\int_0^1 (2 - r\cos\theta - r\sin\theta)\sqrt{3}\, r\,dr\,d\theta$$

$$\int_0^{2\pi}\int_0^1 2\sqrt{3}r - \sqrt{3}r^2\cos\theta - \sqrt{3}r^2\sin\theta \, dr\,d\theta$$

$$\sqrt{3}r^2 - \frac{\sqrt{3}}{3}r^3\cos\theta - \frac{\sqrt{3}}{3}r^3\sin\theta \Big|_0^1 = \sqrt{3} - \frac{\sqrt{3}}{3}\cos\theta - \frac{\sqrt{3}}{3}\sin\theta$$

$$\sqrt{3}\int_0^{2\pi} \left(1 - \frac{\cos\theta}{3} - \frac{\sin\theta}{3}\right) d\theta$$

$$\theta + \frac{\sin\theta}{3} - \frac{\cos\theta}{3} \Big|_0^{2\pi} = 2\pi - \frac{2}{3}$$