Quiz for lecture 20.

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Section: 23

1. Find an equation for the tangent plane to the parametric surface.

$$x=v^2,\ y=u+v,\ z=u^2.$$

at the point $(1,2,1)$.

$$v=u=\pm 1.$$

$$u+v=2$$

$$u=v=1.$$

$$r=v^2\vec{i}+(u+v)\vec{j}+u^2\vec{k}$$

$$r_u=0\vec{i}+\vec{j}+2u\vec{k}$$

$$r_v=2v\vec{i}+\vec{j}+0\vec{k}$$

$$r_u(1,1)=0\vec{i}+\vec{j}+2\vec{k}$$

$$r_v(1,1)=2\vec{i}+\vec{j}+0\vec{k}$$

$$r_u=\langle 0,1,2\rangle$$

$$r_v=\langle 2,1,0\rangle.$$

$$r_u\times r_v=\begin{vmatrix}\vec{i}&\vec{j}&\vec{k}\\0&1&2\\2&1&0\end{vmatrix}=-2\vec{i}+4\vec{j}-2\vec{k}$$

$$N=\langle -2,4,-2\rangle.$$

$$-2(x-1)+4(y-2)-2(z-1)=0.$$

$$x-2y+z+2=0.$$

2. Evaluate the surface integral

$$\iint_S z\,dS.$$

where S is the triangular region with vertices $(2,0,0), (0,2,0), (0,0,2)$.

$$x+y+z=2$$

$$z=2-x-y$$

$$\frac{\partial z}{\partial x}=\frac{\partial z}{\partial y}=-1.$$

$$dS=\sqrt{1+\left(\frac{\partial z}{\partial x}\right)^2+\left(\frac{\partial z}{\partial y}\right)^2}\,dA=\sqrt{3}\,dx\,dy$$

$$\sqrt{3}\int_0^2\int_0^2 (2-x-y)\,dx\,dy$$

Inner Loop: $\left[2x-\frac{x^2}{2}-xy\right]_0^2$

$$=4-2-2y$$

$$=2-2y$$

Outer Loop: $\left[2y-y^2\right]_0^2 \times\sqrt{3}$

$$=0.$$