"QUIZ" for Lecture 20

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E-MAIL SCANNED .pdf OF COMPLETED QUIZ to DrZcalc3@gmail.com (Attachment: q20FirstLast.pdf) ASAP BUT NO LATER THAN Nov. 16, 8:00pm

1. Find an equation for the tangent plane to the parametric surface

$$x = v^2 \quad , \quad y = u + v \quad , \quad z = u^2 \quad ,$$

at the point $(1, 2, 1)$. Simplify as much as you can!

We need to find a vector normal to the surface at $(1,2,1)$. For that, we can find its rate of change (partial derivatives) w.r.t. u and v and take their cross product:

$r = v^2\hat{i} + (u+v)\hat{j} + u^2\hat{k} \rightarrow r_u = 0\hat{i} + \hat{j} + 2u\hat{k} \quad r_v = 2v\hat{i} + \hat{j} + 0\hat{k}$

Find the (u,v) point at which $x=1$, $y=2$, and $z=1$

$1 = v^2 \rightarrow v = 1$; $1 = u^2 \rightarrow u = 1$; $u+v = 1+1 = 2$ ✓ $\rightarrow (u,v) = (1,1)$

Plug $(1,1)$ into r_u and r_v:

$r_u(1,1) = 0\hat{i} + \hat{j} + 2\hat{k}$, $r_v(1,1) = 2\hat{i} + \hat{j} + 0\hat{k}$

Find their cross product:

$$r_u(1,1) \times r_v(1,1) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 1 & 2 \\ 2 & 1 & 0 \end{vmatrix} = \hat{i}(0-2) - \hat{j}(0-4) + \hat{k}(0-2) = -2\hat{i} + 4\hat{j} - 2\hat{k}$$

Now, use the point $(1,2,1)$ and the normal vector to get the equation of the plane:

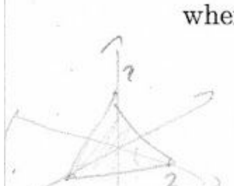
$(-2)(x-1) + 4(y-2) - 2(z-1) = 0 \rightarrow -2x + 2 + 4y - 8 - 2z + 2 = 0 \rightarrow -2x + 4y - 2z = 4 \rightarrow$

$\rightarrow \boxed{x - 2y + z = -2}$

2. Evaluate the surface integral

$$\iint_S z\, dS \quad ,$$

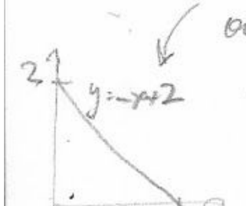
where S is the triangular region with vertices $(2, 0, 0), (0, 2, 0), (0, 0, 2)$.



The triangular region can be seen as a plane $z = 2 - x - y$ in the first octet, so, we can use $dS = \sqrt{1 + (\frac{\partial z}{\partial x})^2 + (\frac{\partial z}{\partial y})^2}\, dA$ to find dS:

$dS = \sqrt{1 + (-1)^2 + (-1)^2}\, dA = \sqrt{3}\, dy\, dx$

The projection onto the xy-plane has bounds $0 \le y \le -x+2$ and $0 \le x \le 2$, so, our new integral is:



$\iint_S z\, dS = \int_0^2 \int_0^{-x+2} 2 - x - y\, (\sqrt{3})\, dy\, dx = \sqrt{3} \int_0^2 \int_0^{-x+2} 2 - x - y\, dy\, dx =$

$= \sqrt{3} \int_0^2 (2y - xy - \frac{y^2}{2}) \Big|_0^{-x+2} dx = \sqrt{3} \int_0^2 2(-x+2) - x(-x+2) - \frac{(-x+2)^2}{2} dx =$

$= \sqrt{3} \int_0^2 -2x + 4 + x^2 - 2x - \frac{x^2 - 4x + 4}{2} dx = \sqrt{3} \int_0^2 -4x + 4 + x^2 - \frac{x^2}{2} + 2x - 2\, dx =$

$$= \sqrt{3}\int_0^2 \frac{x^2}{2} - 2x + 2\,d\varphi = \sqrt{3}\left(\frac{x^3}{6} - x^2 + 2x\right)\Big|_0^2 = \sqrt{3}\left(\frac{8}{6} - 4 + 4\right) = \frac{8\sqrt{3}}{6} =$$

$$= \boxed{\frac{4\sqrt{3}}{3}}$$