

"QUIZ" for Lecture 20

NAME: (print!) Fady Besada Section: 22

E-MAIL SCANNED .pdf OF COMPLETED QUIZ to DrZcalc3@gmail.com (Attachment: q20FirstLast.pdf) ASAP BUT NO LATER THAN Nov. 16, 8:00pm

1. Find an equation for the tangent plane to the parametric surface

$$x = v^2, \quad y = u + v, \quad z = u^2,$$

at the point (1, 2, 1). Simplify as much as you can!

$$r = x\mathbf{i} + y\mathbf{j} + z\mathbf{k} \Rightarrow r = v^2\mathbf{j} + (u+v)\mathbf{j} + u^2\mathbf{k}$$

$$r_u = x_u\mathbf{i} + y_u\mathbf{j} + z_u\mathbf{k} \Rightarrow r_u = \mathbf{j} + 2u\mathbf{k}$$

$$r_v = x_v\mathbf{i} + y_v\mathbf{j} + z_v\mathbf{k} \Rightarrow r_v = 2v\mathbf{i} + \mathbf{j}$$

$$r_u(1,1) = \mathbf{j} + 2\mathbf{k}, \quad r_v(1,1) = 2\mathbf{i} + \mathbf{j}$$

$$r_u \times r_v = \langle -2, 4, -2 \rangle$$

$$\boxed{x - 2y + z = -2}$$

2. Evaluate the surface integral

$$\iint_S z \, dS,$$

where S is the triangular region with vertices (2, 0, 0), (0, 2, 0), (0, 0, 2).

$$x + y + z = 2$$

$$z = 2 - x - y$$

$$\int_0^2 \int_0^{2-x} \sqrt{3} \, dy \, dx = \boxed{2\sqrt{3}}$$

$$\iint_S f(x, y, z) \, dS = \iint_D f(x, y, g(x, y)) \sqrt{1 + g_x^2 + g_y^2} \, dx \, dy$$

$$g(x, y) = 2 - x - y \Rightarrow g_x = -1, \quad g_y = -1$$

$$\iint_S z \, dS = \iint_D (2 - x - y) \sqrt{3} \, dy \, dx$$

$$D = \{ (x, y) \mid x \geq 0, y \geq 0, x + y \leq 2 \}$$

$$D = \{ (x, y) \mid 0 \leq x \leq 2, 0 \leq y \leq 2 - x \}$$

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Evaluate the surface integral $\int \int_S \mathbf{F} \cdot d\mathbf{S}$ for the given vector field \mathbf{F} and oriented surface S .

$$\mathbf{F}(x, y, z) = \langle xy, yz, zx \rangle,$$

and S is the part of the paraboloid $z = 1 - x^2 - y^2$ that lies above the square $0 \leq x \leq 1, 0 \leq y \leq 1$ and has upward orientation.

$$\mathbf{F} = \langle P, Q, R \rangle$$

$$\iint_S \mathbf{F} \cdot d\mathbf{s} = \iint_D \left(-P \frac{dz}{dx} - Q \frac{dz}{dy} + R \right) dA$$

$$P = xy, \quad Q = yz, \quad R = zx$$

$$\iint_D (-xy(-2x) - yz(-2y) + xz) dA$$

$$\iint_D (2x^2y + 2y^2z + xz) dA$$

$$\int_0^1 \int_0^1 (2x^2y + 2y^2(1 - x^2 - y^2) + x(1 - x^2 - y^2)) dA = \boxed{\frac{83}{180}}$$