"QUIZ" for Lecture 20

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E-MAIL SCANNED .pdf OF COMPLETED QUIZ to DrZcalc3@gmail.com (Attachment: q20FirstLast.pdf) ASAP BUT NO LATER THAN Nov. 16, 8:00pm

1. Find an equation for the tangent plane to the parametric surface

$$x = v^2 \quad , \quad y = u + v \quad , \quad z = u^2 \quad ,$$

at the point $(1, 2, 1)$. Simplify as much as you can!

@ point $(1,2,1)$

$v = 1$

$u = 1$

$r = \langle v^2, u+v, u^2 \rangle$

$r_u = \langle 0, 1, 2u \rangle = \langle 0, 1, 2 \rangle$

$r_v = \langle 2v, 1, 0 \rangle \quad \langle 2, 1, 0 \rangle$

$r_u \times r_v = \langle -2, 4, -2 \rangle$

$-2(x-1) + 4(y-2) - 2(z-1) = 0$

$-2x + 2 + 4y - 8 - 2z + 2 = 0$

$-2x + 4y - 2z = 4 \longrightarrow \boxed{x - 2y + z = -2}$

2. Evaluate the surface integral

$$\iint_S z \, dS \quad ,$$

where S is the triangular region with vertices $(2, 0, 0), (0, 2, 0), (0, 0, 2)$.

A B C

$\vec{AB} = \langle -2, 2, 0 \rangle$

$\vec{AC} = \langle -2, 0, 2 \rangle$

$\vec{AB} \times \vec{AC} = \vec{N} = \langle 4, 4, 4 \rangle$

$4(x-2) + 4(y) + 4z = 0$

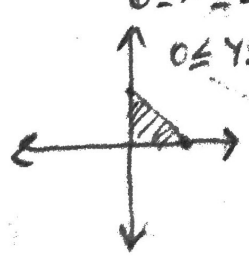
$x + y + z = 2$

$g(x,y) = 2 - x - y$

$y = 2 - x$

$0 \le x \le 2$

$0 \le y \le 2 - x$



$$\iint_S z \, ds = \iint_D 2 - x - y \sqrt{1 + (-1)^2 + (-1)^2} \, dA$$

$$= \sqrt{3} \int_0^2 \int_0^{2-x} 2 - x - y \, dy \, dx = \sqrt{3} \int_0^2 2y - xy - \frac{y^2}{2} \Big|_0^{2-x}$$

$$= \sqrt{3} \int_0^2 4 - 2x - 2x + x^2 - \frac{(4 - 4x + x^2)}{2} dx$$

$$= \sqrt{3} \int_0^2 2 - 2x - \frac{x^2}{2} \, dx$$

$$= \sqrt{3} \left(2x - x^2 - x^3/6\right) \Big|_0^2$$

$$= \sqrt{3} \left(4 - 4 - 4/3\right)$$

$$= -4\frac{\sqrt{3}}{3}$$