Yash Khangura "Quiz" for Lecture 20 Section 24

1. Find an equation for the tangent plane to the parametric surface at the point (1, 2, 1)

$x = v^2,\ y = u + v,\ z = u^2 \qquad u = 1,\ v = 1$

$r = v^2 i + (u+v) j + u^2 k$

$r_u = 0i + 1j + 2u\,k = \langle 0, 1, 2 \rangle$ $= \langle -2, 4, -2 \rangle$

$r_v = 2vi + 1j + 0k = \langle 2, 1, 0 \rangle$

$-2(x-1) + 4(y-2) - 2(z-1) = 0$

$-2x + 2 + 4y - 8 - 2z + 2 = 0$

$-2x + 4y - 2z = 4 \Rightarrow -x + 2y - z = 2$

2. Evaluate the surface integral where S is the triangular region with vertices (2, 0, 0), (0, 2, 0), (0, 0, 2)

$\iint_S z\, dS \qquad \frac{x}{2} + \frac{y}{2} + \frac{z}{2} = 1 \Rightarrow x + y + z = 2$

$z = 2 - x - y$

$\iint_D z\sqrt{(-1)^2 + (-1)^2 + 1}\, dA = \sqrt{3}\iint_D z\, dA;\ 0 \le x \le 2,\ 0 \le y \le 2 - x$

$\sqrt{3}\int_0^2 \int_0^{2-x} z\, dy\,dx$

???

Yash Khangura "Quiz" for lecture 22 Section 24

Evaluate the surface integral $\iint_S F \cdot dS$ for the given vector field $F(x,y,z) = \langle xy, yz, zx \rangle$ and oriented surface S, where S is the part of the paraboloid $z = 1 - x^2 - y^2$ that lies above the square $0 \le x \le 1$, $0 \le y \le 1$ and has an upward orientation.

$$\iint_D -xy(-2x) - yz(-2y) + xz \, dA = \iint_D 2x^2y + z(2y^2 + x) \, dA$$

$$= \iint_D 2x^2y + (1 - x^2 - y^2)(2y^2 + x) \, dA = \iint_D 2x^2y + 2y^2 + x - 2x^2y^2 - x^3 - 2y^4 - xy^2 \, dA$$

$$= \int_0^1 \int_0^1 2x^2y + 2y^2 + x - 2x^2y^2 - x^3 - 2y^4 - xy^2 \, dx\,dy$$

$$= \int_0^1 \frac{2}{3}x^3y + 2xy^2 + \frac{x^2}{2} - \frac{2}{3}x^3y^2 - \frac{x^4}{4} - 2xy^4 - \frac{1}{2}x^2y^2 \Big|_0^1 \, dy$$

$$= \int_0^1 \left(\frac{2}{3}y + 2y^2 + \frac{1}{2} - \frac{2}{3}y^2 - \frac{1}{4} - 2y^4 - \frac{1}{2}y^2\right) - (0) \, dy$$

$$= \int_0^1 \frac{5}{6}y^2 + \frac{2}{3}y - 2y^4 + \frac{1}{4} \, dy = \frac{5}{18}y^3 + \frac{1}{3}y^2 - \frac{2}{5}y^5 + \frac{1}{4}y \Big|_0^1$$

$$= \frac{5}{18} + \frac{1}{3} - \frac{2}{5} + \frac{1}{4} = \frac{11}{18} - \frac{3}{20} = \frac{220 - 54}{360}$$

$$= \frac{166}{360} = \frac{83}{180}$$