A Problem from a Previous Final:

Find an equation for the tangent plane to the parametric surface

$x = u^2 \quad y = u + v \quad z = v^2$ @ the point $(1, 2, 1)$

$\vec{r} = u^2\hat{\imath} + (u+v)\hat{\jmath} + v^2\hat{k}$

$r_u = 2u\hat{\imath} + 1\hat{\jmath} + 0\hat{k}$

$r_v = 0\hat{\imath} + 1\hat{\jmath} + 2v\hat{k}$

$$r_u \times r_v = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 2u & 1 & 0 \\ 0 & 1 & 2v \end{vmatrix}$$

$r_u \times r_v = (2v - 0)\hat{\imath} - (4uv - 0)\hat{\jmath} + (2u - 0)\hat{k}$

$r_u \times r_v = 2v\hat{\imath} - 4uv\hat{\jmath} + 2u\hat{k} \Rightarrow N = \langle 2v, -4uv, 2u \rangle$

Find (u,v):

$x = u^2 = 1 \Rightarrow u = 1$

$y = u + v = 2$

$z = v^2 = 1 \Rightarrow v = 1$

$(u,v) = (1,1)$

N @ $(u,v) = (1,1)$

$N = \langle 2(1), -4(1)(1), 2(1) \rangle = \langle 2, -4, 2 \rangle$

Plug into equation:

$a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$

$(2)(x - 1) + (4)(y - 2) + (2)(z - 1) = 0$

$2x - 2 + 4y - 8 + 2z - 2 = 0$

$2x + 4y + 2z = 12$

Divide by 2 to simplify:

$x + 2y + z = 6$

2. $\iint_S z\,dS$ where S is a Δ region w/ vertices $\underset{A}{(2,0,0)}$, $\underset{B}{(0,2,0)}$ & $\underset{C}{(0,0,2)}$

$\vec{AB} = \langle 0-0, 2-0, 0-2 \rangle = \langle 0, 2, -2 \rangle$

$\vec{BC} = \langle 2-0, 0-2, 0-0 \rangle = \langle 2, -2, 0 \rangle$

$$AB \times BC = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 0 & 2 & -2 \\ 2 & -2 & 0 \end{vmatrix}$$

$AB \times BC = (0-4)\hat{\imath} - (0+4)\hat{\jmath} + (0-4)\hat{k}$

$= -4\hat{\imath} - 4\hat{\jmath} - 4\hat{k}$

$a(x-x_0) + b(y-y_0) + c(z-z_0) = 0$

$(-4)(x-2) + (-4)(y-0) + (-4)(z-0) = 0$

$-4x + 8 - 4y - 4z = 0$

$x + y + z = 2 \Rightarrow z = 2 - x - y$ *

$\iint_S F \cdot dS$

$\cancel{F = \langle xy, yz, zx \rangle}$

paraboloid: $z = 1 - x^2 - y^2$

$0 \le x \le 1 \quad 0 \le y \le 1$

$$\iint_S F \cdot dS = \iint_D \left(-P \frac{dg}{dx} - Q \frac{dg}{dy} + R\right) dA$$

$$= \int_{y=0}^{1} \int_{x=0}^{1} \left(xy(-2x) - (yz)(-2y) + zx \right) dA$$

$$= \int_{y=0}^{1} \int_{x=0}^{1} \left(-2x^2y + 2xy^2 + zx \right) dA$$

inner int:

$$\int_{x=0}^{1} \left(-2x^2y + 2xy^2 + zx\right) dx$$

$$= \left[-\frac{2}{3}x^3y + x^2y^2 + \frac{1}{2}zx^2 \right] \Bigg|_0^1 = -\frac{2}{3}y + y^2 + \frac{1}{2}z$$

$$\int_{y=0}^{1} \left(-\frac{2}{3}y + y^2 + \frac{1}{2}z\right) dy = \left[-\frac{1}{3}y^2 + \frac{1}{3}y^3 + \frac{1}{2}zy \right] \Bigg|_0^1 = -\frac{1}{3} + \frac{1}{3} + \frac{1}{2}z$$