Quiz for Lecture 20

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Section 22
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1、Find an equation for the tangent plane to the parametric surface

$$x = v^2, \quad y = u+v, \quad z = u^2$$

at the point $(1,2,1)$. simplify as much as you can!

$v^2 = 1 \quad u+v = 2 \quad u^2 = 1$

$v = 1 \quad u = 1$

$r_u = \langle 0, 1, 2u \rangle = \langle 0, 1, 2 \rangle$

$r_v = \langle 2v, 1, 0 \rangle = \langle 2, 1, 0 \rangle$

normal vector $= \langle 0,1,2 \rangle \times \langle 2,1,0 \rangle$

$= \langle -2, 4, -2 \rangle$

$-2(x-1) + 4(y-2) - 2(z-1) = 0$

$-x+1 + 2y - 4 - z + 1 = 0$

$\boxed{z = -x+2y-2}$.

2、Evaluate the surface integral

$$\iint_S z \, dS$$

where S is the triangular region with vertices $(2,0,0)$ $(0,2,0)$ $(0,0,2)$

$x+y+z=2$

~~$x+y+z=2$~~

$z = 2-x-y$

$z-x-y=0$

$x+y=2$.

$x \geq 0 \quad y \geq 0$.

$\{0 \leq x \leq 2, 0 \leq y \leq 2-x\}$.

$dS = \sqrt{1^2 + (-1)^2 + (-1)^2} = \sqrt{3}\,dx\,dy$

$\int_0^2 \int_0^{2-x} (2-x-y)\sqrt{3}\,dy\,dx = \frac{4}{3}\sqrt{3}$.

$\boxed{\text{Ans: } \frac{4}{3}\sqrt{3}.}$

Quiz for Lecture 22

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Evaluate the surface integral $\iint_S F \cdot ds$ for the given vector field F and oriented surface S.

~~and S is th~~ $F(x,y,z) = \langle xy, yz, zx \rangle$

and S is the part of the paraboloid $z = 1 - x^2 - y^2$ that lies above the square $0 \le x \le 1$ $0 \le y \le 1$ and has upward orientation.

$$\iint_S F \cdot ds = \left(-P\frac{dg}{dx} - Q\frac{dg}{dy} + R\right) dA$$

$P = xy \quad Q = yz \quad R = zx$

$g = 1 - x^2 - y^2$

$\frac{dg}{dx} = -2x$

$\frac{dg}{dy} = -2y$.

$$\iint_S F \cdot ds = (+2x^2y + 2y^2z + zx)\, dA.$$

$$= (2x^2y + 2y^2(1-x^2-y^2) + (1-x^2-y^2)x)\, dA$$

$\{(x,y) \mid 0 \le x \le 1,\ 0 \le y \le 1\}$.

$$\int_0^1\int_0^1 (2x^2y + 2y^2(1-x^2-y^2) + (1-x^2-y^2)x)\, dx\, dy$$

$$= \frac{83}{180}.$$