Yash Khangura "QUIZ" for Lecture 2 Section 2.4

1. Determine whether the two vectors are orthogonal and if not, whether the angle between them is acute or obtuse.

a. $\langle 1, 1, 1\rangle$, $\langle 3, -2, -1\rangle$

$u = \langle 1, 1, 1\rangle \quad v = \langle 3, -2, -1\rangle$

$u \times v = (1\cdot -1 - -2\cdot 1)i + (3\cdot 1 - 1\cdot -1)j + (1\cdot -7 - 3\cdot 1)k$

$= i + 4j - 5k = \langle 1, 4, -5\rangle$

$u \cdot (u \times v) = \langle 1, 1, 1\rangle \cdot \langle 1, 4, -5\rangle = 1\cdot 1 + 1\cdot 4 + 1\cdot -5 = 1 + 4 - 5 = 0$

$v \cdot (u \times v) = \langle 3, -2, -1\rangle \cdot \langle 1, 4, -5\rangle = 3\cdot 1 + -2\cdot 4 + -1\cdot -5 = 3 - 8 + 5 = 0$

The vectors $u = \langle 1, 1, 1\rangle$ and $v = \langle 3, -2, -1\rangle$ are orthogonal because both $u \cdot (u \times v)$ and $v \cdot (u \times v)$ equal 0.

b. $u = \langle 4, 3\rangle$, $v = \langle 2, -4\rangle$

$\cos\theta = \dfrac{u \cdot v}{\|u\|\,\|v\|}$ $\quad \|u\| = \sqrt{4^2 + 3^2} = 5$ $\quad u \cdot v = 4\cdot 2 + 3\cdot -4 = -4$

$\|v\| = \sqrt{2^2 + -4^2} = \sqrt{20} = 2\sqrt{5}$

$\cos\theta = \dfrac{-4}{10\sqrt{5}} = \dfrac{-2}{5\sqrt{5}}$

$\theta = \cos^{-1}\left(\dfrac{-2}{5\sqrt{5}}\right) = 1.7506 \text{ radians} \approx 100.3°$

The vectors $u = \langle 4, 3\rangle$ and $v = \langle 2, -4\rangle$ are not orthogonal. The angle between them is about 100.3°, so it is obtuse.

2. Calculate $v \times w$, if $v = \langle 0, 1, -1\rangle$, $w = \langle 1, -1, 0\rangle$

$$v \times w = \begin{vmatrix} i & j & k \\ 0 & 1 & -1 \\ 1 & -1 & 0 \end{vmatrix} = (1\cdot 0 - -1\cdot -1)i + (-1\cdot -1 - 0\cdot 0)j + (0\cdot -1 - 1\cdot 1)k = -i - j - k$$

$= \langle -1, -1, -1\rangle$