

"QUIZ" for Lecture 19

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All

NAME: (print!) \_\_\_\_\_ Section: \_\_\_\_\_

**E-MAIL SCANNED .pdf OF COMPLETED QUIZ to DrZcalc3@gmail.com (Attachment: q19FirstLast.pdf) ASAP BUT NO LATER THAN Nov. 12, 8:00pm**

1.

Determine whether or not the vector field

$$F(x, y, z) = y^2 z^3 \mathbf{i} + 2xyz^3 \mathbf{j} + 3xy^2 z^2 \mathbf{k}$$

is conservative. If it is conservative, find a function  $f$  such that  $\mathbf{F} = \nabla f$ .

i	j	k
d/dx	d/dy	d/dz
$y^2 z^3$	$2xyz^3$	$3xy^2 z^2$

$$i (d/dy * 3xy^2 z^2 - d/dz * 2xyz^3) - j (d/dx * 3xy^2 z^2 - d/dz * y^2 z^3) + k (d/dx * 2xyz^3 - d/dy * y^2 z^3) = 0$$

$$\begin{aligned} f_x &= y^2 z^3 \\ f &= x y^2 z^3 + g(y, z) \\ f_y &= 2xyz^3 \\ 2xyz^3 + g_y &= 2xyz^3 \\ g_y &= 0 \quad g(y, z) = h(z) \\ f &= x y^2 z^3 + h(z) \\ f_z &= 3xy^2 z^2 \\ 3xy^2 z^2 + h'(z) &= 3xy^2 z^2 \\ h'(z) &= 0 \quad h(z) = 0 \\ f &= x y^2 z^3 \end{aligned}$$

2. Show that the line integral

$$\int_C 2xy^2 dx + 2x^2 y dy \quad ,$$

is independent of the path  $C$ , and evaluate it if  $C$  is *any* path from  $(1, 0)$  to  $(0, 1)$ .

$$\begin{aligned} f_x &= 2xy^2 \\ f &= x^2 y^2 + g(y) \\ f_y &= 2x^2 y \\ g_y &= 0 \\ g(y) &= 0 \\ f &= x^2 y^2 \\ f(1, 0) - f(0, 1) &= 0 \end{aligned}$$