"QUIZ" for Lecture 19

NAME: (print!) SAI EMBAR **Section:** 23

E-MAIL SCANNED .pdf OF COMPLETED QUIZ to DrZcalc3@gmail.com (Attachment: q19FirstLast.pdf) ASAP BUT NO LATER THAN Nov. 12, 8:00pm

1.

Determine whether or not the vector field

$$F(x, y, z) = y^2z^3\,\mathbf{i} + 2xyz^3\,\mathbf{j} + 3xy^2z^2\,\mathbf{k}$$

is conservative. If it is conservative, find a function f such that $\mathbf{F} = \nabla f$.

$F_1 = y^2z^3$, $F_2 = 2xyz^3$, $F_3 = 3xy^2z^2$

$\frac{\partial F_1}{\partial y} = \frac{\partial F_2}{\partial x} = 2yz^3$

$\frac{\partial F_1}{\partial z} = \frac{\partial F_3}{\partial x} = 3y^2z^2$

$\frac{\partial F_2}{\partial z} = \frac{\partial F_3}{\partial y} = 6xyz^2$

$\frac{\partial f}{\partial x} = F_1$

$\frac{\partial f}{\partial x} = y^2z^3$

$f(x,y,z) = \int y^2z^3\,dx = xy^2z^3 + g(y,z)$

$\frac{\partial f}{\partial y} = F_2$

$\frac{\partial f}{\partial y} = 2xyz^3$, $2xyz^3 + g_y(y,z) = 2xyz^3$

$\to g_y(y,z) = 0$

$\int 0\,dy = 0 + h(z)$

$f(x,y,z) = xy^2z^3 + h(z)$

$\frac{\partial f}{\partial z} = F_3$

$\frac{\partial f}{\partial z} = 3xy^2z^2$

$3xy^2z^2 + h'(z) = 3xy^2z^2$

$h'(z) = 0$

$h(z) = \int 0\,dz = 0 + C$

$\boxed{f(x,y,z) = xy^2z^3 + C}$

2. Show that the line integral

$$\int_C 2xy^2\,dx + 2x^2y\,dy \quad ,$$

is independent of the path C, and evaluate it if C is *any* path from $(1, 0)$ to $(0, 1)$.

$F_1 = 2xy^2$, $F_2 = 2x^2y$

$F_1(y) = 4xy$ $(F_2)_x = 4xy$

$F_1(y) = (F_2)_x$.

$f(x,y) = \int 2xy^2\,dx = x^2y^2 + g(y)$

$\frac{\partial f}{\partial y} = 2x^2y + g'(y)$

$\frac{\partial f}{\partial y} = 2x^2y$

$2x^2y + g'(y) = 2x^2y$

$f(x,y) = x^2y^2$

$\to$ $f(\text{end}) - f(\text{start})$

$f(0,1) - f(1,0) = 0^2\cdot 1 - 1^2\cdot 0 = \boxed{0}$