

"QUIZ" for Lecture 19

NAME: (print!) Prathik Wlla Section: _____

E-MAIL SCANNED .pdf OF COMPLETED QUIZ to DrZcalc3@gmail.com (Attachment: q19FirstLast.pdf) ASAP BUT NO LATER THAN Nov. 12, 8:00pm

1.

Determine whether or not the vector field

$$F(x, y, z) = y^2 z^3 \mathbf{i} + 2xyz^3 \mathbf{j} + 3xy^2 z^2 \mathbf{k}$$

is conservative. If it is conservative, find a function f such that $\mathbf{F} = \nabla f$.

$\begin{array}{ccc} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ y^2 z^3 & 2xyz^3 & 3xy^2 z^2 \end{array}$	$\Rightarrow \text{Curl } F = \langle 0, 0, 0 \rangle$
Vector field is conservative! $f = xy^2 z^3$	

$$\nabla f = \langle f_x, f_y, f_z \rangle$$

$$f_x = y^2 z^3, f_y = 2xyz^3, f_z = 3xy^2 z^2$$

$$f = xy^2 z^3 + h(y, z)$$

$$f_y = 2xyz^3 + h_y \Rightarrow h_y = 0$$

$$f_z = 3xy^2 z^2 + h_z \Rightarrow h_z = 0$$

2. Show that the line integral

$$\int_C 2xy^2 dx + 2x^2 y dy, \quad ,$$

is independent of the path C , and evaluate it if C is any path from $(1, 0)$ to $(0, 1)$.

$$F = \langle 2xy^2, 2x^2 y \rangle$$

$$f_x = 2xy^2, f_y = 2x^2 y$$

$$\int f_x dx = x^2 y^2 + h(y)$$

$$f_y = 2xy^2 + h_y$$

$$f = x^2 y^2$$

$\begin{array}{cc} \mathbf{i} & \mathbf{j} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} \\ 2xy^2 & 2x^2 y \end{array}$	$\Rightarrow \text{Curl}(F) = \langle 0, 0, 0 \rangle$
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$$f(0,1) - f(1,0) = 0^2 \cdot 1^2 - 1^2 \cdot 0^2$$

$$= \boxed{0}$$