"QUIZ" for Lecture 19

NAME: (print!) Orion Kress-Sanfilippo Section: 22

E-MAIL SCANNED .pdf OF COMPLETED QUIZ to DrZcalc3@gmail.com (Attachment: q19FirstLast.pdf) ASAP BUT NO LATER THAN Nov. 12, 8:00pm

1.

Determine whether or not the vector field

$$F(x,y,z) = y^2z^3\,\mathbf{i} + 2xyz^3\,\mathbf{j} + 3xy^2z^2\,\mathbf{k}$$

is conservative. If it is conservative, find a function f such that $\mathbf{F} = \nabla f$.

$F = \langle y^2z^3, 2xyz^3, 3xy^2z^2 \rangle$

$\boxed{f(x,y,z) = xy^2z^3}$

$\nabla \times F = \det \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ y^2z^3 & 2xyz^3 & 3xy^2z^2 \end{vmatrix}$

$\text{curl}(F) = (6xyz^2 - 6xyz^2)\hat{i} - (3y^2z^2 - 3y^2z^2)\hat{j} + (2yz^3 - 2yz^3)\hat{k}$

$= 0$ ✓

2. Show that the line integral

$$\int_C 2xy^2\,dx + 2x^2y\,dy \;, = \int \nabla f\, dr$$

$y = f(x)$

$\boxed{\text{Given any (nice) } r}$

is independent of the path C, and evaluate it if C is *any* path from $(1,0)$ to $(0,1)$.

By Fund. Thm of Line Integrals: $\boxed{\int \nabla f\, dr = f(a) - f(b)}$

$\nabla f = \langle 2xy^2, 2x^2y \rangle =$

✓ Check Conservative:

$\det \begin{vmatrix} \hat{i} & \hat{j} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} \\ 2xy^2 & 2x^2y \end{vmatrix} = 4xy - 4xy$

$= 0$ ✓

$a = (1,0)$

$b = (0,1)$

$f(x,y) = x^2y^2$

$f(a) = 0 \quad f(b) = 0 \Rightarrow \boxed{= 0}$