

"QUIZ" for Lecture 19

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E-MAIL SCANNED .pdf OF COMPLETED QUIZ to DrZcalc3@gmail.com (Attachment: q19FirstLast.pdf) ASAP BUT NO LATER THAN Nov. 12, 8:00pm

1.

Determine whether or not the vector field

$$F(x, y, z) = y^2 z^3 \mathbf{i} + 2xyz^3 \mathbf{j} + 3xy^2 z^2 \mathbf{k}$$

is conservative. If it is conservative, find a function  $f$  such that  $\mathbf{F} = \nabla f$ .

$$\begin{aligned} \text{Curl}(F) &= \left\langle \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right\rangle \times \langle y^2 z^3, 2xyz^3, 3xy^2 z^2 \rangle = \\ &\left\langle \frac{\partial}{\partial y} 3xy^2 z^2 - \frac{\partial}{\partial z} 2xyz^3, \frac{\partial}{\partial x} 3xy^2 z^2 - \frac{\partial}{\partial z} y^2 z^3, \frac{\partial}{\partial x} 2xyz^3 - \frac{\partial}{\partial y} y^2 z^3 \right\rangle = \\ &\langle 6xyz^2 - 6xyz^2, 3y^2 z^2 - 3y^2 z^2, 2yz^3 - 2yz^3 \rangle = 0; \text{ Yes its conservative} \end{aligned}$$

$$\begin{aligned} \int f_x dx &= \int y^2 z^3 dx = xy^2 z^3 + g(y, z) \\ \int f_y dy &= \int 2xyz^3 dy = xy^2 z^3 + g(x, z) \\ \int f_z dz &= \int 3xy^2 z^2 dz = xy^2 z^3 + g(x, y) \end{aligned} \quad \hat{f} = xy^2 z^3$$

2. Show that the line integral

$$\int_C 2xy^2 dx + 2x^2 y dy, \quad P \quad Q$$

is independent of the path  $C$ , and evaluate it if  $C$  is any path from  $(1, 0)$  to  $(0, 1)$ .

$$\int_C x^2 y^2 dr = 0 \quad \text{because} \quad f(P) = f(Q)$$

and the

integrand is conservative