Quiz for lecture 19

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Section: 23

1. Determine whether or not the vector field

$$F(x,y,z) = y^2z^3 i + 2xyz^3 j + 3xy^2z^2 k$$

is conservative. If it is conservative, find a function f such that $F = \nabla f$.

$$\cancel{\frac{\partial}{\partial z}\frac{\partial}{\partial y} y^2z^3 = 6yz^2}$$

$$\cancel{\frac{\partial}{\partial z}\frac{\partial}{\partial x} 2xyz^3 = 6yz^2}$$

$$\cancel{\frac{\partial}{\partial y}\frac{\partial}{\partial x} 3xy^2z^2 = 6yz^2}.$$

$$\text{Curl}(F) = i\left(\frac{\partial}{\partial y}3xy^2z^2 - \frac{\partial}{\partial z}2xyz^3\right) - j\left(\frac{\partial}{\partial x}3xy^2z^2 - \frac{\partial}{\partial z}y^2z^3\right) + k\left(\frac{\partial}{\partial x}2xyz^3 - \frac{\partial}{\partial y}y^2z^3\right) = 0$$

~~They're the same~~ so it is conservative.

$$f = \int y^2z^3\,dx = xy^2z^3 + g(y) + h(z)$$

$$f = \int 2xyz^3\,dy = xy^2z^3 + h(z)$$

$$f = \int 3xy^2z^2\,dz = xy^2z^3$$

$$\therefore f = xy^2z^3$$

2. Show that the line integral

$$\int_C 2xy^2 dx + 2x^2 y dy$$

is independent of the path C. and evaluate it if C. is any path from (1,0) to (0,1)

$$\frac{\partial}{\partial y} 2xy^2 = 4xy$$

$$\frac{\partial}{\partial x} 2x^2y = 4xy$$

They are the same.

$\therefore$ It is conservative

and it is independent of the path C.

Assume $x = t$, $y = 1-t$, $0 \le t \le 1$.

$$f = \int 2xy^2 dx = x^2y^2 + g(y)$$

$$f = \int 2x^2 y dy = x^2y^2. \qquad r(t) = \langle t, 1-t \rangle$$

The path is

$$f(0,1) - f(1,0)$$

$$= 0 - 0$$

$$= 0.$$