"QUIZ" for Lecture 19

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E-MAIL SCANNED .pdf OF COMPLETED QUIZ to DrZcalc3@gmail.com (Attachment: q19FirstLast.pdf) ASAP BUT NO LATER THAN Nov. 12, 8:00pm

1.

Determine whether or not the vector field

$$F(x, y, z) = y^2z^3\,\mathbf{i} + 2xyz^3\,\mathbf{j} + 3xy^2z^2\,\mathbf{k}$$

is conservative. If it is conservative, find a function f such that $\mathbf{F} = \nabla f$.

$\frac{\partial F_1}{\partial y} = 2yz^3 = \frac{\partial F_2}{\partial x} = 2yz^3$, $\frac{\partial F_2}{\partial z} = 3z^2\,2xy = \frac{\partial F_3}{\partial y}$ $2y\,3xz^2$ $\frac{\partial F_3}{\partial x} = 3y^2z^2 = \frac{\partial F_1}{\partial x} = 3y^2z^2$

$\therefore$ The vector field is conservative.

$\int y^2z^3\,dx = xy^2z^3 + g(y,z)$

$g(y,z) = \frac{\partial}{\partial y}\left(xy^2z^3 + g(y,z)\right) = 2xyz^3$

$2xyz^3 + g(y,z) = 2xyz^3$

$g(y,z) = 0 \rightarrow g(y,z) = h(z)$

$\rightarrow xy^2z^3 + h(z)$

$\frac{\partial}{\partial z}\left(xy^2z^3 + h(z)\right) = 3xy^2z^2 + h'(z) = 3xy^2z^2$

$h'(z) = 0 \rightarrow h(z) = C$

potential function: $f(x,y,z) = \boxed{3xy^2z^2 + C}$

2. Show that the line integral

$$\int_C 2xy^2\,dx + 2x^2y\,dy \quad ,$$

is independent of the path C, and evaluate it if C is *any* path from $(1, 0)$ to $(0, 1)$.

$F_1 = 2xy^2 \quad F_2 = 2x^2y$

$\frac{\partial F_1}{\partial y} = 4xy = \frac{\partial F_2}{\partial x} = 4xy \xrightarrow{\therefore}$ conservative.

$\int 2xy^2\,dx = x^2y^2 + g(y)$

$\frac{\partial}{\partial y}\left(x^2y^2 + g(y)\right) = 2x^2y + g'(y) = 2x^2y$

$\rightarrow g(y) = 0$

$f(1,0) - f(0,1) = \boxed{0}$