"QUIZ" for Lecture 19

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E-MAIL SCANNED .pdf OF COMPLETED QUIZ to DrZcalc3@gmail.com (Attachment: q19FirstLast.pdf) ASAP BUT NO LATER THAN Nov. 12, 8:00pm

1.

Determine whether or not the vector field

$$F(x,y,z) = \overset{P}{y^2z^3}\,\mathbf{i} + \overset{Q}{2xyz^3}\,\mathbf{j} + \overset{R}{3xy^2z^2}\,\mathbf{k}$$

is conservative. If it is conservative, find a function f such that $\mathbf{F} = \nabla f$.

The vector field is conservative is $\frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x}$:

$\frac{\partial P}{\partial y} = 2 \cdot y \cdot z^3 \quad \frac{\partial Q}{\partial x} = 2y \cdot z^3$ ✓

Since it is conservative, we can find its potential function.
First, we integrate P w.r.t. x, since $f_x = P$:

$\int y^2z^3 dx = xy^2z^3 + g(y,z)$, where $g(y,z)$ is like an arbitrary constant.
Derive the result w.r.t y and set it equal to Q:

$2xyz^3 + g_y = 2xyz^3 \Rightarrow g_y = 0$, so, $g(y,z) = 0 \rightarrow f(x,y,z) = xy^2z^3 + h(z)$

Derive the result w.r.t. z and set it equal to R:

$3xy^2z^2 + h_z = 3xy^2z^2 \rightarrow h_z = 0$, so, $h(z) = 0 \rightarrow \boxed{f(x,y,z) = xy^2z^3}$

2. Show that the line integral

$$\int_C \overset{P}{2xy^2}\,dx + \overset{Q}{2x^2y}\,dy \quad ,$$

is independent of the path C, and evaluate it if C is *any* path from $(1,0)$ to $(0,1)$.

It is independent of the path C if $\frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x}$:

$\frac{\partial P}{\partial y} = 4xy \quad \frac{\partial Q}{\partial x} = 4xy$ ✓

We can now find its potential function. First, integrate P w.r.t. x, since $P = f_x$:

$\int 2xy^2 dx = x^2y^2 + g(y)$, where $g(y)$ is like an arbitrary constant.
Derive the result w.r.t. y and set it equal to Q:

$2x^2y + g_y = 2x^2y \Rightarrow g_y = 0$, so, $g(y) = 0 \Rightarrow f(x,y) = x^2y^2$

We can find the line integral over the path C using its endpoints, (1,0) and (0,1), since f is independent of the path

$f(0,1) - f(1,0) = (0^2)(1^2) - (1^2)(0^2) = \boxed{0}$