

"QUIZ" for Lecture 19

NAME: (print!) Gillian Mulvey Section: _____

E-MAIL SCANNED .pdf OF COMPLETED QUIZ to DrZcalc3@gmail.com (Attachment: q19FirstLast.pdf) ASAP BUT NO LATER THAN Nov. 12, 8:00pm

1.

Determine whether or not the vector field

$$F(x, y, z) = y^2 z^3 \mathbf{i} + 2xyz^3 \mathbf{j} + 3xy^2 z^2 \mathbf{k}$$

is conservative. If it is conservative, find a function f such that $F = \nabla f$.

$$\begin{vmatrix} \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ y^2 z^3 & 2xyz^3 & 3xy^2 z^2 \end{vmatrix} = (6xyz^2 - 6xyz^2)\mathbf{i} - (3y^2 z^2 - 3y^2 z^2)\mathbf{j} + (2yz^3 - 2yz^3)\mathbf{k} = 0 \text{ conservative}$$

$$f_x = y^2 z^3 \quad f_y = 2xyz^3 \quad f_z = 3xy^2 z^2$$

$$\int y^2 z^3 dx = xy^2 z^3 + g(y, z)$$

$$f = xy^2 z^3 + 2xz^3 - \frac{xy^4 z^3}{12} + h(z)$$

$$\frac{xy^3 z^3}{3} + g_y(y, z) = 2xyz^3$$

$$g_y = 2xyz^3 - \frac{xy^3 z^3}{3}$$

$$h'(z) = 3xy^2 z^2$$

$$h(z) = xy^2 z^3$$

$$f = 2xyz^3 + 2xz^3 - \frac{xy^4 z^3}{12}$$

2. Show that the line integral

$$\int_C 2xy^2 dx + 2x^2 y dy$$

is independent of the path C , and evaluate it if C is any path from $(1, 0)$ to $(0, 1)$.

$$\int 2xy^2 dx + 2x^2 y dy =$$

$$4xy = 4xy \quad \checkmark$$

$$\frac{df}{dx} = 2xy^2 \quad \frac{df}{dy} = 2x^2 y$$

$$f = x^2 y^2$$

$$f = x^2 y^2 + g(y)$$

$$f(1, 0) - f(0, 1) = 0 - 0 = 0$$

$$\frac{d}{dy} x^2 y^2 + g(y) = 2x^2 y$$

$$2yx^2 + g'(y) = 2x^2 y$$

$$g(y) = 0$$