“QUIZ” for Lecture 19

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E-MAIL SCANNED .pdf OF COMPLETED QUIZ to DrZcalc3@gmail.com (Attachment: q19FirstLast.pdf) ASAP BUT NO LATER THAN Nov. 12, 8:00pm

1.

Determine whether or not the vector field

$$F(x, y, z) = y^2z^3\,\mathbf{i} + 2xyz^3\,\mathbf{j} + 3xy^2z^2\,\mathbf{k}$$

is conservative. If it is conservative, find a function f such that $\mathbf{F} = \nabla f$.

$\rightarrow \begin{vmatrix} i & j & k \\ d/dx & d/dy & d/dz \\ y^2z^3 & 2xyz^3 & 3xy^2z^2 \end{vmatrix}$ shows that $\text{curl}(F) = \langle 0,0,0 \rangle$

$\rightarrow \nabla f = \langle f_x, f_y, f_z \rangle$

$\rightarrow f_x = y^2z^3,\ f_y = 2xyz^3,\ f_z = 3xy^2z^2$

$\rightarrow f = xy^2z^3 + h(y,z)$

$\rightarrow f_y = 2xyz^3 + h_y \Rightarrow (h_y = 0)$

$\rightarrow f_z = 3xy^2z^2 + h_z \Rightarrow (h_z = 0)$

$\rightarrow$ Vector field is conservative; $\boxed{f = xy^2z^3}$

2. Show that the line integral

$$\int_C 2xy^2\,dx + 2x^2y\,dy \quad ,$$

is independent of the path C, and evaluate it if C is *any* path from $(1, 0)$ to $(0, 1)$.

$\rightarrow F = \langle 2xy^2, 2x^2y \rangle$

$\rightarrow f_x = 2xy^2,\ f_y = 2x^2y$

$\rightarrow \begin{vmatrix} i & j \\ d/dx & d/dy \\ 2xy^2 & 2x^2y \end{vmatrix} \Rightarrow \text{curl}(F) = \langle 0,0,0 \rangle$

$\rightarrow \int f_x\,dx = x^2y^2 + h(y)$

$\rightarrow f_y = 2xy^2 + h_y$

$\rightarrow f = x^2y^2$

$\rightarrow f(0,1) - f(1,0) = \boxed{0}$