

"QUIZ" for Lecture 19

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E-MAIL SCANNED .pdf OF COMPLETED QUIZ to DrZcalc3@gmail.com (Attachment: q19FirstLast.pdf) ASAP BUT NO LATER THAN Nov. 12, 8:00pm

1.

Determine whether or not the vector field

$$F(x, y, z) = y^2 z^3 \mathbf{i} + 2xyz^3 \mathbf{j} + 3xy^2 z^2 \mathbf{k}$$

is conservative. If it is conservative, find a function  $f$  such that  $F = \nabla f$ .

$$\text{curl}(F) = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ y^2 z^3 & 2xz^3 & 3xy^2 z^2 \end{vmatrix}$$

$$= \mathbf{i}(6xyz^3 - 6xyz^3) - \mathbf{j}(3y^2 z^2 - 3y^2 z^2) + \mathbf{k}(2yz^3 - 2yz^3)$$

$= 0$ , conservative

$$f(x, y, z) = xy^2 z^3 + g(y, z)$$

$$2xyz^3 = f_y(x, y, z) = 2xyz^3 + g_y(y, z)$$

$$g_y(y, z) = 0$$

$$h(z) = 0$$

$$f(x, y, z) = xy^2 z^3$$

2. Show that the line integral

$$\int_C 2xy^2 dx + 2x^2 y dy$$

is independent of the path  $C$ , and evaluate it if  $C$  is any path from  $(1, 0)$  to  $(0, 1)$ .

$$\left. \begin{aligned} \frac{\partial}{\partial y} 2xy^2 &= 4xy \\ \frac{\partial}{\partial x} 2x^2 y &= 4xy \end{aligned} \right\} = \text{independent of path } C$$

$$\int_C 2xy^2 dx + 2x^2 y dy = \int \nabla f \cdot dr$$

$$= f(0, 1) - f(1, 0)$$

$$f(x, y) = x^2 y^2 + h(y)$$

$$\left. \begin{aligned} 2x^2 y &= f_y(x, y) = 2x^2 y + h'(y) \\ h'(y) &= 0 \quad h(y) = 0 \end{aligned} \right\} f(x, y) = x^2 y^2$$

$$= \boxed{0}$$