

Quiz 19.

$$Q1. F(x, y, z) = y^2 z^3 \mathbf{i} + 2xy z^3 \mathbf{j} + 3xy^2 z^2 \mathbf{k}$$

$$\text{Answer: } f_x = y^2 z^3 \quad f_y = 2xy z^3 \quad f_z = 3xy^2 z^2$$

$$\frac{d}{dy} (P) = zy z^3 \quad \frac{d}{dx} (Q) = 2yz^3$$

$$\frac{d}{dz} (P) = 3y^2 z^2 \quad \frac{d}{dx} (R) = 3y^2 z^2$$

\therefore it is conservative.

$$\therefore f_x = y^2 z^3$$

$$f = \int f_x dx = xy^2 z^3 + g(y, z)$$

$$\frac{d}{dy} f = 2xy z^3 + g_y(y, z) = 2xy z^3$$

$$\therefore g_y(y, z) = 0$$

$$\therefore f = xy^2 z^3 + h(z)$$

$$\frac{d}{dz} f = 3xy^2 z^2 + h'(z) = 3xy^2 z^2$$

$$\therefore h'(z) = 0$$

$$\therefore f = xy^2 z^3$$

Q2. $\int_C zxy^2 dx + 2x^2 y dy$ C is any path from (1,0) to (0,1)

$$f = \int zxy^2 dx = x^2 y^2 + g(y) \quad \frac{d}{dy} P = 2yzx$$

$$\frac{d}{dy} f = 2yx^2 + g_y = 2x^2 y$$

$$\therefore g_y = 0$$

$$f = (x^2 y^2)$$

$$\frac{d}{dx} Q = 2xzy$$

\therefore it is conservative.

$$f(0,1) - f(1,0) = 0 - 0 = 0$$

