Quiz for Lecture 19

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1. Determine whether or not the vector field

$$F(x,y,z) = y^2z^3 i + 2xyz^3 j + 3xy^2z^2 k$$

is conservative. If it is conservative, find a function f such that $F = \nabla f$.

$$\begin{vmatrix} i & j & k \\ \frac{d}{dx} & \frac{d}{dy} & \frac{d}{dz} \\ y^2z^3 & 2xyz^3 & 3xy^2z^2 \end{vmatrix}$$

$$i\left(\frac{d}{dy}(3xy^2z^2) - \frac{d}{dz}(2xyz^3)\right) - j\left(\frac{d}{dx}(3xy^2z^2) - \frac{d}{dz}(y^2z^3)\right) + k\left(\frac{d}{dx}(2xyz^3) - \frac{d}{dy}(y^2z^3)\right)$$

$$= i(6xyz^2 - 6xyz^2) - j(3y^2z^2 - 3y^2z^2) + k(2yz^3 - 2yz^3).$$

$$= \langle 0,0,0\rangle = 0.$$

F is conservative.

$f\_x = y^2z^3 \quad f\_y = 2xyz^3 \quad f\_z = 3xy^2z^2.$

$f = xy^2z^3 + g(y,z)$

$f\_y = 2xyz^3 + g'(y) = 2xyz^3.$

$g'(y) = 0 \quad g(y) = 0$

$f = xy^2z^3 + h(z)$

$f\_z = 3xy^2z^2 + h'(z) = 3xy^2z^2.$

$h'(z) = 0 \quad h(z) = 0$

$f = xy^2z^3.$

$\boxed{\text{Ans: } f = xy^2z^3.}$

2. show that the line integral

$$\int_C 2xy^2dx + 2x^2ydy$$

is independent of the path C, and evaluate it if C is any path from (1,0) to (0,1).

partA:

~~curl~~ $\frac{d}{dy}(2xy^2) = 4xy$

$\frac{d}{dx}(2x^2y) = 4xy$

$4xy = 4xy$

~~Thet~~

Ans: line integral is independent of the path C, because $4xy = 4xy$.

partB:

method A:

Because of independent of the path C (conservative), we can use the fundamental theorem of line integrals to calculate this integrals.

$$\int_C 2xy^2dx + 2x^2ydy$$

$$= 2y^2 + 2x^2 \Big|_{(1,0)}^{(0,1)}$$

$$= (2\times1^2 + 2\times0^2) - (2\times0^2 + 2\times1^2)$$

$$= 2 - 2$$

$$= 0$$

Ans: 0.

$\boxed{\text{Final Ans: 0.}}$

method B

line $(1,0) + t(-1,1) = (1-t, t)$.

$x = 1-t \quad y = t$

$dx = -dt \quad dy = dt$

$0 \le t \le 1$

$$\int_0^1 2(1-t)t^2 \cdot -dt + 2(1-t)^2 t \cdot dt$$

$$= \int_0^1 2t + 4t^3 - 6t^2 \, dt$$

$$= t + t^4 - 2t^3 \Big|_0^1$$

$$= 1 + 1 - 2 - 0$$

$$= 2 - 2$$

$$= 0$$

Ans: 0