

q18 Rahul Paleja

Section: 22

- ① Let C be the line segment from $(0,1)$ to $(2,3)$
Find $\int_C xy \, ds$

Parametric Representation of Line Segment:

$$0 \leq t \leq 1 \rightarrow \langle 0, 1 \rangle + t \langle 2, 2 \rangle \quad x(t) = 2t \quad y(t) = 1 + 2t$$

$$ds = \sqrt{x'(t)^2 + y'(t)^2} = \sqrt{2^2 + 2^2} = \sqrt{8} \, dt = 2\sqrt{2} \, dt$$

$$\int_0^1 2(t)(1+2t) 2\sqrt{2} \, dt =$$

$$2\sqrt{2} \int_0^1 2t + 4t^2 \, dt = 2\sqrt{2} \int_0^1 2(t + 2t^2) \, dt$$

$$= 4\sqrt{2} \int_0^1 (t + 2t^2) \, dt = 4\sqrt{2} \left[\frac{t^2}{2} \right]_0^1 + \frac{2t^3}{3} \Big|_0^1$$

$$= \frac{7 \cdot 2^{3/2}}{3} = \boxed{6.599}$$

- ② Evaluate $\int_C xy^2 \, dx + x^2y \, dy$ where C is
 $x = t^2, y = t^3, 0 \leq t \leq 1$
 $dx = 2t \, dt \quad dy = 3t^2 \, dt$

$$\int_0^1 t^2 (t^3)^2 \cdot 2t \, dt + (t^2)^2 (t^3) \cdot (3t^2) \, dt$$

$$= \int_0^1 2t^9 \, dt + 3t^9 \, dt$$

$$= \int_0^1 5t^9 \, dt = 5 \left[\frac{t^{10}}{10} \right]_0^1 = 5 \cdot \frac{1^{10}}{10} =$$

$$= \boxed{\frac{1}{2}}$$