"QUIZ" for Lecture 18

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E-MAIL SCANNED .pdf OF COMPLETED QUIZ to DrZcalc3@gmail.com (Attachment: q18FirstLast.pdf) ASAP BUT NO LATER THAN Nov. 9, 8:00pm

1. Let C be the line segment from $(0,1)$ to $(2,3)$, find $\int_C xy\,ds$.

The line segment can be parametrically represented as:

$x = t$, $y = t+1$, $0 \le t \le 2$

The formula for ds is $\sqrt{x'(t)^2 + y'(t)^2}$. So,

$ds = \sqrt{(t)'^2 + (t+1)'^2} = \sqrt{1^2+1^2} = \sqrt{2}$

Represent the integrated function using the parametrizations above:

$xy = (t)(t+1) = t^2 + t$

Plug them into the integral:

$$\int_C xy\,ds = \int_0^2 (t^2+t)(\sqrt{2})\,dt = \sqrt{2}\int_0^2 t^2+t\,dt = \sqrt{2}\left(\frac{t^3}{3} + \frac{t^2}{2}\right)\Big|_0^2 =$$

$$= \sqrt{2}\left(\frac{8}{3} + \frac{4}{2}\right) = \sqrt{2}\left(\frac{14}{3}\right) = \boxed{\frac{14\sqrt{2}}{3}}$$

2. Evaluate

$$\int_C xy^2\,dx + x^2y\,dy\,,$$

where C is $x : t^2$, $y = t^3$, $0 \le t \le 1$.

First, find dx (or $\frac{dx}{dt}$) and dy (or $\frac{dy}{dt}$):

$dx = 2t\,dt$; $dy = 3t^2\,dt$

Represent the integrated functions as functions of t:

$xy^2 = (t^2)(t^6) = t^8$, $x^2y = (t^4)(t^3) = t^7$

Plug the results into the integral:

$$\int_C xy^2\,dx + x^2y\,dy = \int_0^1 t^8 + t^7\,dt = \frac{t^9}{9} + \frac{t^8}{8}\Big|_0^1 =$$

$$= \frac{1}{9} + \frac{1}{8} = \boxed{\frac{17}{72}}$$