

Quiz for Lecture 18

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section 22
10.10.2020.

1. Let C be the line segment from $(0, 1)$ to $(2, 3)$
find $\int_C xy \, ds$.

$$\text{line: } (0, 1) + t(2, 2) = (2t, 2t+1) \quad 0 \leq t \leq 1$$

$$x = 2t \quad y = 2t+1$$

$$dx = 2dt \quad dy = 2dt$$

$$ds = 2\sqrt{2} \, dt$$

$$\int_0^1 2t(2t+1) 2\sqrt{2} \, dt$$

$$= 2\sqrt{2} \int_0^1 4t^2 + 2t \, dt$$

$$= 2\sqrt{2} \times \left(\frac{4}{3}t^3 + t \right) \Big|_0^1$$

$$= 2\sqrt{2} \times \left(\frac{4}{3} + 1 \right)$$

$$= 2\sqrt{2} \times \frac{7}{3}$$

$$= \frac{14}{3}\sqrt{2}$$

$$\text{Ans: } \frac{14}{3}\sqrt{2}$$

2. Evaluate $\int_C xy^2 dx + x^2 y dy$
where C is $x = t^2$, $y = t^3$, $0 \leq t \leq 1$.

$$x = t^2 \quad y = t^3$$

$$dx = 2t dt \quad dy = 3t^2 dt$$

$$\int_0^1 t^2 \cdot (t^3)^2 \cdot 2t dt + (t^2)^2 \cdot t^3 \cdot 3t^2 dt$$

$$= \int_0^1 2t^9 dt + 3t^9 dt$$

$$= \int_0^1 5t^9 dt$$

$$= \frac{5}{10} t^{10} \Big|_0^1$$

$$= \frac{1}{2} (1^{10} - 0)$$

$$= \frac{1}{2} \times 1$$

$$= \frac{1}{2}$$

$$\boxed{\text{Ans: } \frac{1}{2}}$$