"QUIZ" for Lecture 17

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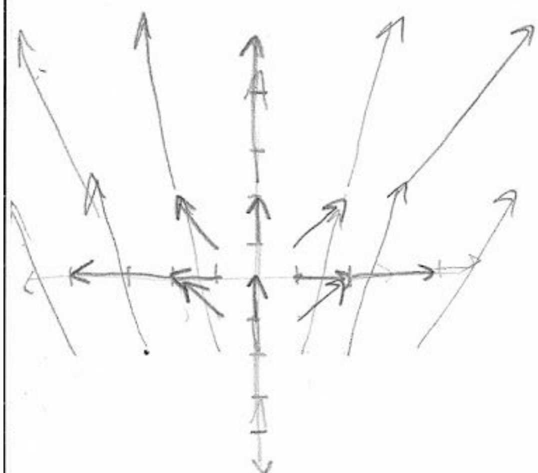
E-MAIL SCANNED .pdf OF COMPLETED QUIZ to DrZcalc3@gmail.com (Attachment: q17FirstLast.pdf) ASAP BUT NO LATER THAN Nov. 5, 8:00pm

1. Sketch the vector planar vector field



$$\mathbf{F} = \langle x, y^2 \rangle \quad .$$

A few example points:

$(0,0) \to F = \langle 0,0 \rangle$

$(1,0) \to F = \langle 1,0 \rangle$ $(-1,0) \to F = \langle -1,0 \rangle$

$(0,1) \to F = \langle 0,1 \rangle$ $(0,-1) \to F = \langle 0,1 \rangle$

$(1,1) \to F = \langle 1,1 \rangle$ $(-1,-1) \to F = \langle -1,1 \rangle$

$(-1,1) \to F = \langle -1,1 \rangle$ $(1,-1) \to F = \langle 1,1 \rangle$

2. Find a potential function for the vector field $\mathbf{F}$

$$\mathbf{F} = \langle y\cos(xy), x\cos(xy) \rangle \quad .$$

First, check that the function is conservative (curl(F) = 0):

$$\nabla \times F = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ y\cos(xy) & x\cos(xy) & 0 \end{vmatrix} = \hat{i}(0-0) - \hat{j}(0-0) + \hat{k}\left(\frac{\partial}{\partial x} x\cos(xy) - \frac{\partial}{\partial y} y\cos(xy)\right) = \hat{k}(0) = 0 \checkmark$$

Because $F = \nabla f$, we know that:

$y\cos(xy) = f_x$ and $x\cos(xy) = f_y$

Let's integrate the first component with respect to x:

$\int y\cos(xy)\,dx = \sin(xy) + g(y) \Rightarrow g(y)$ is to account for the integral of f_y. If we take the result's derivative with respect to y, and set it equal to f_y, we get:

$$x\cos(xy) + g_y = x\cos(xy)$$

$$g_y = 0$$

So, our function is $\boxed{\sin(xy)}$