

Quiz for lecture 16.

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Section: 8:40-10:00 A.M.

1. Compute the Jacobian of the transformation

$$\phi(r, s) = (rs, r+s)$$

$$f = rs \quad f_r = s \quad f_s = r.$$

$$g = r+s \quad g_r = 1 \quad g_s = 1.$$

$$\begin{vmatrix} s & r \\ 1 & 1 \end{vmatrix} = s-r.$$

2. Let $D = \phi(R)$ where $\phi(u, v) = (u+v, v^2)$ and $R = [0, 6] \times [1, 2]$

Calculate $\iint_D y \, dA$

$$f = u+v \quad f_u = 1 \quad f_v = 1$$

$$g = v^2 \quad g_u = 0 \quad g_v = 2v$$

$$\begin{vmatrix} 1 & 1 \\ 0 & 2v \end{vmatrix} = 2v \neq.$$

$$\iint_D y \, dA = \iint_R v^2 \cdot 2v \, dA$$

$$\int_0^6 \int_1^2 2v^3 - v^2 \, dv \, du$$

$$\text{Inner Loop: } \left[\frac{v^4}{2} - \frac{v^3}{3} \right]_1^2 = 8 - \frac{8}{3} - \frac{1}{2} + \frac{1}{3} = \frac{31}{6}$$

$$\text{Outer Loop: } 6 \times \frac{31}{6} = 31$$

$$\int_0^6 \int_1^2 2v^3 \, dv \, du$$

$$\text{Inner Loop: } \left[\frac{v^4}{2} \right]_1^2 = 8 - \frac{1}{2} = \frac{15}{2}$$

$$\text{Outer Loop: } 6 \times \frac{15}{2} = 45.$$

