

"QUIZ" for Lecture 16

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E-MAIL SCANNED .pdf OF COMPLETED QUIZ to DrZcalc3@gmail.com (Attachment: q16FirstLast.pdf) ASAP BUT NO LATER THAN Nov. 2, 8:00pm

1. Compute the Jacobian of the transformation

$$\Phi(r, s) = (rs, r + s)$$

The formula for the Jacobian is:

$$J = \begin{vmatrix} x_r & x_s \\ y_r & y_s \end{vmatrix}$$

So, we first find all of the partial derivatives required:

$$x = rs \rightarrow x_r = s \quad x_s = r$$

$$y = r + s \rightarrow y_r = 1 \quad y_s = 1$$

Plug them in:

$$J = \begin{vmatrix} s & r \\ 1 & 1 \end{vmatrix} = (s \cdot 1) - (r \cdot 1) = \boxed{s - r}$$

2. Let $D = \Phi(R)$ where $\Phi(u, v) = (u + v, v^2)$ and $R = [0, 6] \times [1, 2]$. Calculate

$$\iint_D y \, dA$$

(Note: it is not necessary to compute D).

First, find the Jacobian:

$$J = \begin{vmatrix} x_u & x_v \\ y_u & y_v \end{vmatrix} = \begin{vmatrix} 1 & 1 \\ 0 & 2v \end{vmatrix} = 2v \cdot 1 - 0 = 2v$$

Substitute $y = v^2$ into the integral:

$$\iint_D v^2 \cdot 2v \, du \, dv = \boxed{\iint_D 2v^3 \, du \, dv}$$

Our region R is a rectangle with vertices $(0, 1)$, $(6, 1)$, $(6, 2)$ and $(0, 2)$.

We need to convert each vertex to the new u, v coordinates.