Lecture 16 Attendance Quiz

Problem 1. Compute the Jacobian of the transformation: $\Phi(r,s) = (rs, r+s)$

$x = rs$

$y = r+s$

Jacobian Matrix:

$$\begin{vmatrix} \frac{dx}{dr} & \frac{dx}{ds} \\ \frac{dy}{dr} & \frac{dy}{ds} \end{vmatrix} = \begin{vmatrix} s & r \\ 1 & 1 \end{vmatrix}$$

Jacobian Matrix Determinant: $= \boxed{s-r}$

Problem 2. Let $D = \Phi(R)$ where $\Phi(u,v) = (u+v, v^2)$ and $R = [0,6] \times [1,2]$.

→ calculate $\iint_D y \, dA$

$x = u + v$

$y = v^2$

Jacobian Matrix:

$$\begin{vmatrix} \frac{dx}{du} & \frac{dx}{dv} \\ \frac{dy}{du} & \frac{dy}{dv} \end{vmatrix} = \begin{vmatrix} 1 & 1 \\ 0 & 2v \end{vmatrix}$$

Deteriminant $= 2v$

$R = [0,6] \times [1,2]$

implies...

$0 \le u \le 6$

$1 \le v \le 2$

$$\iint_D y\,dA = \int_0^6 \int_1^2 y \times (\text{Jacobian Determinant})\, dv\,du$$

$$= \int_0^6 \int_1^2 y(2v)\,dv\,du = \int_0^6 \int_1^2 (v^2)(2v)\,dv\,du = \int_0^6 \int_1^2 2v^3\,dv\,du$$

inner integral:

$$2\int_0^2 v^3\,dv = 2\left[\tfrac{1}{4}v^4\right]\Big|_0^2 = 2\left(\tfrac{1}{4}\right)(2)^4 - 0 = 8$$

outer integral:

$$\int_0^6 (8)\,du = [8u]\Big|_0^6 = 8(6) - 0 = 40$$

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