Vash Khangura "Quiz" for Lecture 15 section 24

1. Use polar coordinates to compute the double integral

$\iint_D xy\, dA$ where $D = \{(x,y) \mid x^2 + y^2 \le 1, x \ge 0, y \ge 0\}$

$\int_0^{\pi}\int_{-1}^{1} r^2 \cos\theta \sin\theta \, r\, dr\, d\theta = \int_0^{\pi}\int_{-1}^{1} r^3 \cos\theta \sin\theta \, dr\, d\theta = \int_0^{\pi} \frac{r^4}{4} \cos\theta \sin\theta \Big|_{-1}^{1} d\theta =$

$\int_0^{\pi} \frac{1}{2} \cos\theta \sin\theta \, d\theta = -\frac{1}{2}\int_{-1}^{1} u\, du = \frac{1}{2} \cdot \frac{u^2}{2}\Big|_{-1}^{1} = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$

$u = \cos\theta$

$du = -\sin\theta\, d\theta$

$-du = \sin\theta\, d\theta$

2. Evaluate the iterated integral by converting it to polar coordinates

$\int_0^1 \int_0^{\sqrt{1-y^2}} e^{x^2+y^2}\, dx\, dy = \int_0^1 \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} e^{r^2} r\, d\theta\, dr = \pi \int_0^1 e^{r^2} r\, dr = \pi \cdot \frac{e^{r^2}}{2}\Big|_0^1 = \frac{\pi}{2}(e-1)$

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