"QUIZ" for Lecture 15

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E-MAIL SCANNED .pdf OF COMPLETED QUIZ to DrZcalc3@gmail.com (Attachment: qXFirstLast.pdf) ASAP BUT NO LATER THAN Oct. 29, 8:00pm

1. Use polar coordinates to compute the double integral

$$\int\int_D xy\, dA \quad , \qquad xy \Rightarrow r^2 \cos\theta \sin\theta$$

where

$$D = \{(x,y)\,|\, x^2+y^2 \le 1\, ,\, x \ge 0\, ,\, y \ge 0\} \quad .$$

$$\int_0^{\frac{\pi}{2}}\int_0^1 r^3 \cos\theta\sin\theta\, dr\, d\theta \Rightarrow \int_0^1 r^3 \cos\theta \sin\theta\, dr = \left| \frac{r^4 \cos\theta \sin\theta}{4} \right|_0^1 = \frac{\cos\theta\sin\theta}{4}$$

$$\frac{1}{4}\int_0^{\frac{\pi}{2}} \cos\theta\sin\theta\, d\theta = \left| -\frac{\cos^2\theta}{2} \right|_0^{\frac{\pi}{2}} \cdot \frac{1}{4} = (0) - \left(-\frac{1}{2}\right) = \left(\frac{1}{2}\right)\frac{1}{4}$$

$$= \boxed{\frac{1}{8}}$$

2. Evaluate the iterated integral by converting it to polar coordinates

$$\int_0^1 \int_0^{\sqrt{1-y^2}} e^{x^2+y^2}\, dx\, dy \quad . \qquad e^{r^2\cos^2\theta + r^2\sin^2\theta} = e^{r^2(1)}$$

Note: The previous version had a typo ($dy\, dx$ instead of $dx\, dy$, that made it nonsense). I thank Yidi "Wendy" Weng for pointing it out (and see won a dolllar).

$$\int_0^{\frac{\pi}{2}}\int_0^1 e^{r^2} r\, dr\, d\theta \Rightarrow \int_0^1 e^{r^2} r\, dr = \left| 2r^2 e^{r^2} + e^{r^2} \right|_0^1 = (3e) - (1)$$

$$\int_0^{\frac{\pi}{2}} (3e-1)\, d\theta = \left| 3e\theta - \theta \right|_0^{\frac{\pi}{2}} = \left(\frac{3\pi}{2}e - \frac{\pi}{2}\right) - (0) = \boxed{\frac{3\pi}{2}e - \frac{\pi}{2}}$$