"QUIZ" for Lecture 15

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E-MAIL SCANNED .pdf OF COMPLETED QUIZ to DrZcalc3@gmail.com (Attachment: qXFirstLast.pdf) ASAP BUT NO LATER THAN Oct. 29, 8:00pm

1. Use polar coordinates to compute the double integral

$$\int\int_D xy\, dA \quad ,$$

where

$$D = \{(x,y) \,|\, x^2 + y^2 \leq 1 \,,\, x \geq 0 \,,\, y \geq 0\} \quad .$$

$y^2 = 1 - x^2$

$y = \sqrt{(1-x^2)}$

$D = \{(r,\theta) \,|\, 0 \leq \theta \leq \frac{\pi}{2},\ 0 \leq r \leq 1\}$

$$\int_0^{\frac{\pi}{2}}\int_0^1 r^3 \cos\theta \sin\theta \, dr\, d\theta$$

$$= \frac{r^4 \cos\theta \sin\theta}{4}\Big|_0^1 = \frac{\cos\theta \sin\theta}{4}$$

$$\int_0^{\frac{\pi}{2}} \frac{\cos\theta \sin\theta}{4}\, d\theta = \int_0^{\pi/2} \frac{u}{4}\, du = \frac{u^2}{8} = \frac{\sin^2\theta}{8}\Big|_0^{\pi/2} = \boxed{\frac{1}{8}}$$

$u = \sin\theta$

$du = \cos\theta\, d\theta$

2. Evaluate the iterated integral by converting it to polar coordinates

$$\int_0^1 \int_0^{\sqrt{1-y^2}} e^{x^2+y^2}\, dx\, dy \quad .$$

Note: The previous version had a typo ($dy\,dx$ instead of $dx\,dy$, that made it nonsense). I thank Yidi "Wendy" Weng for pointing it out (and see won a dolllar).

$D = \{(x,y) \,|\, 0 \leq y \leq 1,\ 0 \leq x \leq \sqrt{1-y^2}\}$

$D = \{(r,\theta) \,|\, 0 \leq \theta \leq \frac{\pi}{2},\ 0 \leq r \leq 1\}$

$$\int_0^{\pi/2}\int_0^1 e^{(r\cos\theta)^2+(r\sin\theta)^2}\, r\, dr\, d\theta$$

$$= \frac{r^2}{2} e^{(r\cos\theta)^2+(r\sin\theta)^2}\Big|_0^1 = \frac{e^{(\cos\theta)^2+(\sin\theta)^2}}{2} = \frac{e}{2}$$

$$\int_0^{\pi/2} \frac{e}{2}\, d\theta = \frac{e}{2}\theta\Big|_0^{\pi/2} = \boxed{\frac{\pi e}{4}}$$