"QUIZ" for Lecture 15

NAME: (print!) SAI EMBAR **Section:** 23

E-MAIL SCANNED .pdf OF COMPLETED QUIZ to DrZcalc3@gmail.com (Attachment: qXFirstLast.pdf) ASAP BUT NO LATER THAN Oct. 29, 8:00pm

1. Use polar coordinates to compute the double integral

$$\int\int_D xy\,dA \quad ,$$

where

$$D = \{(x,y)\,|\,x^2+y^2 \le 1,\, x \ge 0,\, y \ge 0\} \quad .$$

$\{(r,\theta)\,|\,0 \le \theta \le \pi/2,\ 0 \le r \le 1\}$

$\int_0^{\pi/2}\int_0^1 (r\cos\theta)(r\sin\theta)\,r\,dr\,d\theta$

$= \int_0^{\pi/2} \cos\theta\sin\theta\,d\theta \left(\int_0^1 r^3\,dr\right)$

$= \int_0^{\pi/2} \frac{\sin 2\theta}{2}\,d\theta \left(\int_0^1 r^3\,dr\right) = \frac{-\cos 2\theta}{4}\Big|_0^{\pi/2}\left(\frac{r^4}{4}\Big|_0^1\right)$

$= \frac{-\cos\pi - (-\cos 0)}{4}\cdot\frac{1}{4} = \boxed{\frac{1}{8}}$

2. Evaluate the iterated integral by converting it to polar coordinates

$$\int_0^1 \int_0^{\sqrt{1-y^2}} e^{x^2+y^2}\,dx\,dy \quad .$$

Note: The previous version had a typo ($dy\,dx$ instead of $dx\,dy$, that made it nonsense). I thank Yidi "Wendy" Weng for pointing it out (and see won a dolllar).

$\{(r,\theta)\,|\,0 \le \theta \le \pi/2,\ 0 \le r \le 1\}$

$\int_0^{\pi/2}\int_0^1 e^{r^2} r\,dr\,d\theta$

$\int_0^1 re^{r^2}\,dr$, u-sub

$\int_0^1 \frac{e^u}{2} = \frac{1}{2}e^u\Big|_0^1 = \frac{e-1}{2}$

$\int_0^{\pi/2} \frac{e-1}{2}\,d\theta = \frac{e-1}{2}\theta\Big|_0^{\pi/2} = \frac{e-1}{2}\cdot\frac{\pi}{2} = \boxed{\frac{(e-1)\pi}{4}}$