"QUIZ" for Lecture 15

NAME: (print!) Prathik Lolla **Section:** _____

E-MAIL SCANNED .pdf OF COMPLETED QUIZ to DrZcalc3@gmail.com (Attachment: qXFirstLast.pdf) ASAP BUT NO LATER THAN Oct. 29, 8:00pm

1. Use polar coordinates to compute the double integral

$$\int\int_D xy\,dA \quad ,$$

where

$$D = \{(x,y)\,|\,x^2+y^2 \le 1\,,\, x \ge 0\,,\, y \ge 0\} \quad .$$

$\{(r,\theta) \mid 0 \le \theta \le \pi/2,\ 0 \le r \le 1\}$

$$\int_0^{\pi/2}\int_0^1 (r\cos\theta)(r\sin\theta)\,r\,dr\,d\theta$$

$$\left(\int_0^{\pi/2} \cos\theta\sin\theta\,d\theta\right)\left(\int_0^1 r^3 d\theta\right)$$

$$= \left(\frac{-\cos 2\theta}{4}\Big|_0^{\pi/2}\right)\left(\frac{r^4}{4}\Big|_0^1\right)$$

$$= \frac{-\cos\pi + \cos 0}{4}$$

$$= \boxed{\frac{1}{8}}$$

2. Evaluate the iterated integral by converting it to polar coordinates

$$\int_0^1 \int_0^{\sqrt{1-y^2}} e^{x^2+y^2}\,dx\,dy \quad .$$

Note: The previous version had a typo ($dy\,dx$ instead of $dx\,dy$, that made it nonsense). I thank Yidi "Wendy" Weng for pointing it out (and see won a dolllar).

$\{(x,y) \mid 0 \le y \le 1,\ 0 \le x \le \sqrt{1-y^2}\}$

$\{(r,\theta) \mid 0 \le \theta \le \pi/2,\ 0 \le r \le 1\}$

$$\int_0^{\pi/2}\int_0^1 e^{r^2} r\,dr\,\theta$$

$$\int_0^1 r e^{r^2} dr$$

$$\int_0^{\pi/2} \frac{e-1}{2}\, d\theta = \frac{e-1}{2}\theta \Big|_0^{\pi/2} = \frac{e-1}{2} \cdot \frac{\pi}{2}$$

$$\frac{(e-1)\pi}{4}$$