“QUIZ” for Lecture 15

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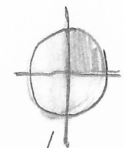
E-MAIL SCANNED .pdf OF COMPLETED QUIZ to DrZcalc3@gmail.com (Attachment: qXFirstLast.pdf) ASAP BUT NO LATER THAN Oct. 29, 8:00pm

1. Use polar coordinates to compute the double integral

$$\int\int_D xy\,dA \quad ,$$



where

$$D = \{(x, y)\,|\,x^2 + y^2 \le 1\,,\ x \ge 0\,,\ y \ge 0\} \quad .$$

$$= \{(r,\theta)\,|\,0 \le \theta \le \tfrac{\pi}{2},\ 0 \le r \le 1\}$$

$$\int_0^{2\pi}\int_0^1 r^3 \sin\theta\cos\theta\,dr\,d\theta = \int_0^{\pi/2} \sin\theta\cos\theta\cdot\left(\frac{1}{4} - 0\right)d\theta = \frac{1}{4}\int_0^{2\pi} \sin\theta\cos\theta\,d\theta$$

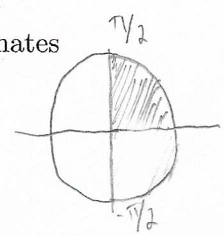
$$= \frac{1}{4}\int_0^{2\pi} \frac{1}{2}\left(\sin(2\theta) + \cancel{\sin(0)}\right)d\theta =$$

$$\frac{1}{8}\left(-\frac{\cos(2\theta)}{2}\right)\Bigg|_0^{\frac{\pi}{2}} = \frac{1}{16}\left(+1 - (-1)\right) = \boxed{\frac{1}{8}}$$

2. Evaluate the iterated integral by converting it to polar coordinates

Semi-circle, rad. 1

$$\int_0^1 \int_0^{\sqrt{1-y^2}} e^{x^2+y^2}\,dx\,dy \quad .$$



Note: The previous version had a typo ($dy\,dx$ instead of $dx\,dy$, that made it nonsense). I thank Yidi "Wendy" Weng for pointing it out (and see won a dolllar).

$$D = \{(x,y)\,|\,0 \le x \le \sqrt{1-y^2},\ 0 \le y \le 1\}$$

$$= \{(r,\theta)\,|\,0 \le \theta \le \frac{\pi}{2},\ 0 \le r \le 1\}$$

$$\int\int_0^1 r e^{r^2}\,dr\,d\theta =$$

$$\int_0^{\pi/2} \frac{e-1}{2}\,d\theta = \boxed{\frac{\pi}{2}\cdot\left(\frac{e-1}{2}\right)}$$