“QUIZ” for Lecture 15

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E-MAIL SCANNED .pdf OF COMPLETED QUIZ to DrZcalc3@gmail.com (Attachment: qXFirstLast.pdf) ASAP BUT NO LATER THAN Oct. 29, 8:00pm

1. Use polar coordinates to compute the double integral

$$\int\int_D xy\,dA \quad ,$$

$\hookrightarrow r\cos\theta\, r\sin\theta$

$= r^2\sin\theta\cos\theta$

$r^2 \frac{\sin\theta}{2}$

where

$$D = \{(x,y)\,|\,x^2+y^2 \le 1\,,\, x \ge 0\,,\, y \ge 0\,\} \quad .$$

$r^2 \le 1$

$r = 1$

$D = \{(r,\theta)\,|\,0 \le \theta \le \frac{\pi}{2}\,,\ 0 \le r \le 1\}$

$\int_0^{\pi/2}\int_0^1 r^2\left(\frac{\sin\theta}{2}\right) r\,dr\,d\theta$

$D = \{(x,y)\,|\,0 \le x \le \sqrt{4-y^2}\,,\ 0 \le y \le \sqrt{4-x^2}\}$

first quadrant D

$\frac{\sin\theta}{2} \cdot \frac{r^3}{3}\Big|_0^1$

$= \frac{\sin\theta}{\cancel{2}} \cdot \frac{1}{3} = \frac{\sin\theta}{3}$

$\int_0^{\pi/2} \frac{\sin\theta}{3}\,d\theta$

$\frac{\sin\frac{\pi}{2}}{3} = \frac{1}{3}$

2. Evaluate the iterated integral by converting it to polar coordinates

$$\int_0^1\int_0^{\sqrt{1-y^2}} e^{x^2+y^2}\,dx\,dy \quad .$$

e^{r^2}

$D = \{(x,y)\,|\,0 \le x \le \sqrt{1-y^2}\,,\ 0 \le y \le 1\}$

$x^2 \le 1-y^2$

$x^2+y^2 \le 1$

$r \le 1$

first quad

$0 \le \theta \le \frac{\pi}{2}$

Note: The previous version had a typo ($dy\,dx$ instead of $dx\,dy$, that made it nonsense). I thank Yidi ”Wendy” Weng for pointing it out (and see won a dolllar).

$D: \{(r,\theta)\,|\,0 \le \theta \le \frac{\pi}{2}\,,\ 0 \le r \le 1\}$

$\int_0^{\pi/2}\int_0^1 e^{r^2}r\,dr\,d\theta$

$\frac{1}{2}e^{r^2}\Big|_0^1 = \frac{1}{2}e - \frac{1}{2} = \frac{1}{2}(e-1)$

$\frac{1}{2}\int_0^{\pi/2} e-1\,d\theta = \frac{1}{2}\cdot\left(\frac{\pi}{2}e - \frac{\pi}{2}\right)$

$= \frac{\pi}{4}(e-1)$