Quiz for Lecture 15.

NAME: Jiahe Lü.

Section: 8:40-10:00 A.M.

1. Use polar coordinates to compute the double integral

$$\iint_D xy\,dA$$

$$D=\{(x,y)\mid x^2+y^2\le 1,\ x\ge 0,\ y\ge 0\}$$

$$r^2\le 1$$

$$0\le r\le 1.$$

$$0\le\theta\le\frac{\pi}{2}$$

$$\int_0^{\frac{\pi}{2}}\int_0^1 r^3\sin\theta\cos\theta\,dr\,d\theta$$

Inner: $\sin\theta\cos\theta\cdot\int_0^1 r^3\,dr$

$$=\sin\theta\cos\theta\cdot\left[\frac{r^4}{4}\right]_0^1$$

$$=\frac{1}{4}\sin\theta\cos\theta$$

Outer: $\frac{1}{4}\int_0^{\frac{\pi}{2}}\sin\theta\cos\theta\,d\theta.$

$$=\frac{1}{4}\cdot\left[\frac{(\sin\theta)^2}{2}\right]_0^{\frac{\pi}{2}}$$

$$=\frac{1}{8}$$

2. Evaluate the iterated integral by converting it to polar coordinates

$$\int_0^1 \int_0^{\sqrt{1-y^2}} e^{x^2+y^2}\,dx\,dy$$

$$D=\{(x,y) \mid 0\le y\le 1,\ 0\le x\le \sqrt{1-y^2}\}$$

$$D=\{(r,\theta) \mid 0\le\theta\le\tfrac{\pi}{2},\ 0\le r\le 1\}$$

$$\int_0^{\frac{\pi}{2}} \int_0^1 e^{r^2}\cdot r\,dr\,d\theta$$

Inner: $\left[\frac{e^{r^2}}{2}\right]_0^1$

$$=\frac{e-1}{2}$$

Outer: $\int_0^{\frac{\pi}{2}} \frac{e-1}{2}\,d\theta$

$$=\frac{\pi}{2}\cdot\frac{e-1}{2}$$

$$=\frac{\pi(e-1)}{4}$$