

"QUIZ" for Lecture 15

NAME: (print!) Jennifer Gonzalez Section: 23

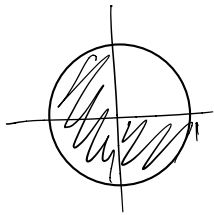
E-MAIL SCANNED .pdf OF COMPLETED QUIZ to DrZcalc3@gmail.com (Attachment: qXFirstLast.pdf) ASAP BUT NO LATER THAN Oct. 29, 8:00pm

1. Use polar coordinates to compute the double integral

$$\iint_D xy \, dA \quad ,$$

where

$$D = \{(x, y) \mid x^2 + y^2 \leq 1, x \geq 0, y \geq 0\} \quad .$$



$$0 \leq r \leq 1$$

$$0 \leq \theta \leq \frac{\pi}{2}$$

$$\int_0^{\pi/2} \int_0^1 r \cos \theta \cdot r \sin \theta \, r \, dr \, d\theta = \int_0^{\pi/2} \int_0^1 r^3 \cos \theta \sin \theta \, dr \, d\theta$$

$$\int_0^1 \cos \theta \sin \theta \, r^3 \, dr = \frac{\cos \theta \sin \theta \, r^4}{4} \Big|_0^1 = \frac{\cos \theta \sin \theta}{4}$$

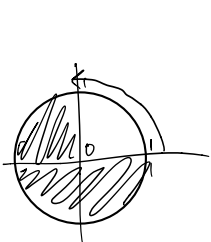
$$\frac{1}{4} \int_0^{\pi/2} \cos \theta \sin \theta \, d\theta \rightarrow \int u \, du = \frac{u^2}{2} \rightarrow \frac{\sin^2 \theta}{2} \Big|_0^{\pi/2}$$

$$= \frac{1}{4} \left(\frac{\sin^2(\pi/2)}{2} - \frac{\sin^2(0)}{2} \right) = \boxed{\frac{\sin^2(\pi/2)}{8}}$$

2. Evaluate the iterated integral by converting it to polar coordinates

$$\int_0^1 \int_0^{\sqrt{1-y^2}} e^{x^2+y^2} \, dx \, dy \quad . = \iint e^r \cdot r \, dr \, d\theta$$

Note: The previous version had a typo ($dy \, dx$ instead of $dx \, dy$, that made it nonsense). I thank Yidi "Wendy" Weng for pointing it out (and see won a dollar).



$$x = \sqrt{1-y^2} \quad x=0$$

$$x^2 = 1-y^2$$

$$x^2+y^2=1$$

$$1 \leq r \leq 3$$

$$0 \leq \theta \leq \pi/2$$

$$\int_0^{\pi/2} \int_1^3 e^r \, r \, dr \, d\theta$$

$$\int_1^3 e^r \, r \, dr = e^r \frac{r^2}{2} \Big|_1^3 = \frac{9e^3}{2} - \frac{e}{2}$$

$$\int_0^{\pi/2} \frac{9e^3 - e}{2} \, d\theta = \frac{9e^3 - e}{2} \left(\theta \Big|_0^{\pi/2} \right) = \boxed{\frac{9e^3 - e}{2} \left(\frac{\pi}{2} \right)}$$