"QUIZ" for Lecture 15

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E-MAIL SCANNED .pdf OF COMPLETED QUIZ to DrZcalc3@gmail.com (Attachment: qXFirstLast.pdf) ASAP BUT NO LATER THAN Oct. 29, 8:00pm

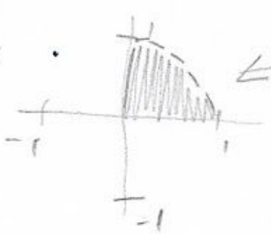
1. Use polar coordinates to compute the double integral

$$\int\int_D xy\,dA \quad ,$$

where

$$D = \{(x,y)\,|\,x^2+y^2 \le 1\,,\, x \ge 0\,,\, y \ge 0\} \quad .$$

$x^2+y^2 \le 1$ represents a circle of radius 1:

D:  radius = 1 $\quad x = r\cos\theta$

$0 \le \theta \le \frac{\pi}{2} \quad y = r\sin\theta$

$$\int_0^{\frac{\pi}{2}}\int_0^1 r^3\cos\theta\sin\theta\,dr\,d\theta = \int_0^{\frac{\pi}{2}}\left(\frac{r^4}{4}\right)\cos\theta\sin\theta\Big|_0^1 d\theta =$$

$$= \int_0^{\frac{\pi}{2}}\frac{\cos\theta\sin\theta}{4}\,d\theta \to \begin{matrix} u = \sin\theta \\ du = \cos\theta\,d\theta \end{matrix} \to \int_0^1 \frac{u}{4}\,du = \frac{u^2}{8}\Big|_0^1 = \boxed{\frac{1}{8}}$$

2. Evaluate the iterated integral by converting it to polar coordinates

$$\int_0^1 \int_0^{\sqrt{1-y^2}} e^{x^2+y^2}\,dx\,dy \quad .$$

Note: The previous version had a typo ($dy\,dx$ instead of $dx\,dy$, that made it nonsense). I thank Yidi "Wendy" Weng for pointing it out (and see won a dolllar).

Region:  radius = 1 $\quad x = r\cos\theta$

$-\frac{\pi}{2} \le \theta \le \frac{\pi}{2} \quad y = r\sin\theta$

$$\int_0^1\int_0^{\frac{\pi}{2}} e^{r^2(\cos^2\theta+\sin^2\theta)}r\,d\theta\,dr = \int_0^1\int_0^{\frac{\pi}{2}} e^{r^2}r\,d\theta\,dr = \int_0^1 \theta e^{r^2}r\Big|_0^{\frac{\pi}{2}}dr = \int_0^1 \pi e^{r^2}r\,dr = \pi\int_0^1 e^{r^2}r\,dr$$

Substitute $u = r^2$, and $du = 2r\,dr$.

$$\pi\int_0^1 e^u\left(\frac{du}{2}\right) = \frac{\pi}{2}\int_0^1 e^u\,du = \frac{\pi}{2}\,e^u\Big|_0^1 = \frac{\pi}{2}(e^1 - e^0) = \boxed{\frac{\pi(e-1)}{2}}$$