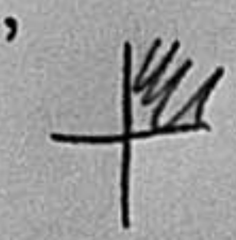
"QUIZ" for Lecture 15

NAME: (print!) Gillian Mulvay **Section:** ______

E-MAIL SCANNED .pdf OF COMPLETED QUIZ to DrZcalc3@gmail.com (Attachment: qXFirstLast.pdf) ASAP BUT NO LATER THAN Oct. 29, 8:00pm

1. Use polar coordinates to compute the double integral

$$\int\int_D xy\, dA \quad ,$$



where

$$D = \{(x, y)\,|\, x^2 + y^2 \leq 1\,,\, x \geq 0\,,\, y \geq 0\} \quad .$$

$r^2 \leq 1$ $\quad r\cos\theta \geq 0 \quad r\sin\theta \geq 0$

$\int_0^{\pi/2}\int_0^1 r^2 \sin\theta \cdot \cos\theta \cdot r\,dr\,d\theta \quad 0 \leq r \leq 1 \quad \cos\theta \geq 0 \quad \sin\theta \geq 0$

$\theta \geq \frac{\pi}{2} \quad \theta \leq 0$

$\int_0^1 r^3 \cdot \sin\theta \cdot \cos\theta = \frac{r^4}{4}\Big|_0^1 = \frac{1}{4}\sin\theta\cos\theta \quad 0 \leq \theta \leq \frac{\pi}{2}$

$\frac{1}{4}\int_0^{\pi/2} \sin\theta\cos\theta\, d\theta = \quad u = \cos\theta \quad du = \sin\theta\, d\theta$

$\frac{1}{4}\int_0^{\pi/2} u\,du = \frac{1}{4}\left(\frac{u^2}{2}\right)\Big|_0^{\pi/2}$

$= \frac{1}{8}\cos^2\theta\Big|_0^{\pi/2} = \frac{1}{8}(\cos^2\frac{\pi}{2} - \cos^2 0) = \frac{1}{8}(0 - 1) = \frac{-1}{8}$

2. Evaluate the iterated integral by converting it to polar coordinates

$$\int_0^1 \int_0^{\sqrt{1-y^2}} e^{x^2+y^2}\, dx\, dy \quad .$$

Note: The previous version had a typo (*dy dx* instead of *dx dy*, that made it nonsense). I thank Yidi "Wendy" Weng for pointing it out (and see won a dolllar).

$0 \leq x \leq \sqrt{1-y^2} \quad 0 \leq y \leq 1 \quad r^2 = x^2 + y^2$

$0 \leq x \leq 1 \quad r = 0 \quad r = 1 - y + (1)^2$

$r = 2 - (1) = 1 \quad 0 \leq r \leq 1$

$\int_0^{\pi/2}\int_0^1 e^{r^2} r\,dr\,d\theta = \int_0^{\pi/2} \frac{1}{2}e^{r^2}\Big|_0^1 = \frac{1}{2}\int_0^{\pi/2} e - 1\, d\theta = \frac{(e-1)\pi}{4}$