"QUIZ" for Lecture 15

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E-MAIL SCANNED .pdf OF COMPLETED QUIZ to DrZcalc3@gmail.com (Attachment: qXFirstLast.pdf) ASAP BUT NO LATER THAN Oct. 29, 8:00pm

1. Use polar coordinates to compute the double integral

$$\int\int_D xy\,dA \quad ,$$

where

$$D = \{(x,y)\,|\,x^2+y^2 \le 1\,,\, x \ge 0\,,\, y \ge 0\} \quad .$$

$dy\,dx = r\,dr\,d\theta$ $\quad x^2+y^2 = r^2$ $\quad x = \sqrt{r^2-y^2}$ $\quad y = \sqrt{r^2-x^2}$

$$\int_0^{\pi}\int_{r\ge 0}^{r\le 1} \sqrt{r^2-y^2}\cdot\sqrt{r^2-x^2}$$

$x = r\cos\theta$

$y = r\sin\theta$

replace

2. Evaluate the iterated integral by converting it to polar coordinates

$$\int_0^1 \int_0^{\sqrt{1-y^2}} e^{x^2+y^2}\,dx\,dy \quad .$$

Note: The previous version had a typo ($dy\,dx$ instead of $dx\,dy$, that made it nonsense). I thank Yidi "Wendy" Weng for pointing it out (and see won a dolllar).

$r = x^2+y^2$ $\quad x = \sqrt{1-y^2}$ $\quad x^2 = 1-y^2$ $\quad x^2+y^2 = 1$

$r = 0 \to 1$

$\theta = 0 \to \pi$

↓

$$\int_0^{\pi} \int_0^{1} e^{r^2} \cdot r \, dr \, d\theta \qquad u = r^2$$

$$du = 2r \, dr$$

$$\frac{1}{2} \int \int e^{u} \, du \, d\theta$$

$$\frac{1}{2} e^{r^2} \Big]_0^1 \qquad \frac{1}{2} e$$

$$\int_0^{\pi} \frac{1}{2} e$$

$$\frac{e\theta}{2} \Big|_0^{\pi}$$

$$= \boxed{\frac{\pi e}{2}}$$