"QUIZ" for Lecture 15

NAME: (print!) Aditya Sivakumar **Section:** 24

E-MAIL SCANNED .pdf OF COMPLETED QUIZ to DrZcalc3@gmail.com (Attachment: qXFirstLast.pdf) ASAP BUT NO LATER THAN Oct. 29, 8:00pm

1. Use polar coordinates to compute the double integral

$$\int\int_D xy\,dA \ ,$$

where

$$D = \{(x,y) \,|\, x^2 + y^2 \le 1,\ x \ge 0,\ y \ge 0\} \ .$$

$u = \sin\theta$
$du = \cos\theta\, d\theta$

$0 \le r \le 1$

$0 \le \theta \le \pi/2$ (x & y are +)

$$\int_0^{\pi/2}\int_0^1 r^3 \sin\theta\cos\theta\, dr\, d\theta =$$

$$\int_0^{\pi/2} \frac{r^4}{4}\sin\theta\cos\theta\, d\theta\Big|_0^1\, d\theta =$$

$$\frac{1}{4}\int_0^{\pi/2} \sin\theta\cos\theta\, d\theta =$$

$$\frac{1}{4}\int_0^1 u\, du =$$

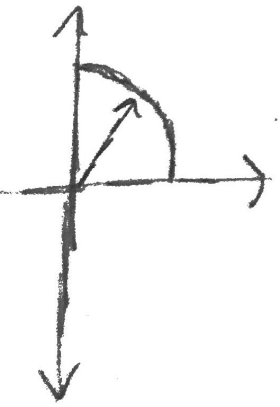
$$\frac{1}{4}\,\frac{u^2}{2}\Big|_0^1 = \boxed{\frac{1}{8}}$$

2. Evaluate the iterated integral by converting it to polar coordinates

$$\int_0^1 \int_0^{\sqrt{1-y^2}} e^{x^2+y^2}\, dx\, dy \ .$$

$0 \le x \le \sqrt{1-y^2}$
$0 \le y \le 1$

Note: The previous version had a typo ($dy\,dx$ instead of $dx\,dy$, that made it nonsense). I thank Yidi "Wendy" Weng for pointing it out (and see won a dolllar).



$$\int_0^{\pi/2}\int_0^1 r e^{r^2}\, dr\, d\theta \rightarrow \int_0^{\pi/2} \frac{e-1}{2}\, d\theta$$

$$= \int_0^{\pi/2} \frac{e^{r^2}}{2}\Big|_0^1\, d\theta = \frac{e-1}{2}\,[\theta]\Big|_0^{\pi/2}$$

$$= \frac{\pi}{4}(e-1)$$

$$\boxed{\frac{\pi}{4}(e-1)}$$