No.

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QUIZ 15

Q1. $\iint_D xy\,dA$

$D=\{(x,y)\mid x^2+y^2\le 1,\ x\ge 0,\ y\ge 0\}$

$x^2+y^2=1$

$x=\sqrt{1-y^2}$

$y=\sqrt{1-x^2}$

$-\sqrt{1-y^2}\le x\le\sqrt{1-y^2}$

$0\le x\le\sqrt{1-y^2}$

$0\le y\le\sqrt{1-y^2}$

$\therefore r^2\le 1$

$r=\pm 1$

r x be negative

$\therefore r=1$

$0\le r\le 1$ $\quad 0\le\theta\le\pi$

$$\int_0^{\pi}\int_0^1 xy\,dA$$

$$=\int_0^{\pi}\int_0^1 r\cos\theta\cdot r\sin\theta\cdot r\,dr\,d\theta$$

$$=\int_0^{\pi}\int_0^1 r^3\cos\theta\cdot\sin\theta\,dr\,d\theta$$

$$=\int_0^{\pi}\left(\cos\theta\cdot\sin\theta\cdot\frac{r^4}{4}\Big|_0^1\right)d\theta$$

$$=\int_0^{\pi}\frac{\cos\theta\cdot\sin\theta}{4}\,d\theta$$

$$=\frac{1}{4}\int_0^{\pi}\cos\theta\cdot\sin\theta\,d\theta$$

$$=\frac{\cos\sin\pi^3}{12}$$

Campus

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Quiz 15

Q2. $\int_0^1 \int_0^{\sqrt{1-y^2}} e^{x^2+y^2}\, dx\, dy$.

$D: \{(x,y) \mid 0 \le x \le \sqrt{1-y^2} \quad 0 \le y \le 1\}$

$x = \sqrt{1-y^2}$

$\therefore 0 \le r \le 1$ (from the graph)

$0 \le \theta \le \frac{\pi}{2}$

$\int_0^{\frac{\pi}{2}} \int_0^1 e^{r^2} \cdot r\, dr\, d\theta$

$\Rightarrow \int_0^{\frac{\pi}{2}} \left(\frac{1}{2} e^{r^2} \Big|_0^1\right) d\theta$

$= \int_0^{\frac{\pi}{2}} \frac{1}{2}(e-1)\, d\theta$

$= \frac{e-1}{2} \cdot \frac{\pi}{2}$

$= \frac{\pi(e-1)}{4}$